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TRAINING IN PROBLEM SOLVING.

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THIS STUDY AIMED AT--(1) CHARACTERIZING PROBLEM-SOLVING PROCESSES OF DIFFERENT INDIVIDUALS AND (2) DETERMINING THE EFFECTS OF TRAINING ON THE SUBJECTS' APPROACHES TO THEIR PROBLEM-SOLVING PERFORMANCE. THE DESIGN INCLUDED 50 NINTH-GRADE STUDENTS AND 50 COLLEGE FRESHMEN DIVIDED INTO EXPERIMENTAL AND CONTROL GROUPS IN WHICH SUBJECTS WERE MATCHED ON THE BASES OF AGE, IQ, AND EDUCATIONAL LEVEL. A PREVIOUSLY DEVELOPED PROBLEM WAS ADMINISTERED TO ALL SUBJECTS. EACH SUBJECT ATTENDED INDIVIDUAL TRAINING SESSIONS EVERY WEEK OVER A 6-MONTH PERIOD FOR A TOTAL OF 20 TO 24 SESSIONS. ANOTHER PREVIOUSLY DEVELOPED PROBLEM WAS ADMINISTERED AFTER TRAINING. STATISTICAL COMPARISONS WERE APPLIED TO THE PERFORMANCE OF THE EXPERIMENTAL AND CONTROL SUBJECTS AT BOTH EDUCATIONAL LEVELS. ACADEMIC GRADES WERE ALSO COMPARED. EVIDENCE SEEMED TO INDICATE THAT TACTICS AND NUMBER OF QUESTIONS ASKED ARE NOT CLEARLY RELATED TO NUMBER OF CORRECT SOLUTIONS TO THE PROBLEMS ADMINISTERED. ALSO, COLLEGE STUDENTS ASKED FEWER IRRELEVANT QUESTIONS THAN HIGH SCHOOL STUDENTS, AND EXPERIMENTAL SUBJECTS DID THE SAME WHEN COMPARED TO CONTROL SUBJECTS. THIS OCCURRENCE WAS ATTRIBUTED TO THEIR INCREASED EXPERIENCE AND KNOWLEDGE. FINALLY, COLLEGE STUDENTS ARE, ON THE WHOLE, BETTER THAN HIGH SCHOOL STUDENTS IN SOLVING PROBLEMS. (HB)

TRAINING IN PROBLEM SOLVING

Cooperative Research Project No. 1449

by

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This report has been signed by those persons who, during the period October 1, 1962, to November 30, 1963, participated in the experiment. Miss Isabel O. Reyes was responsible for the testing of the college students and for performing most of the statistical work involved with the results obtained from these studies. Miss Rose Mary Zacharia administered some of the problems to the high school students and performed part of the computational work. Mr. James B. Erdmann has been very active in the whole project. He not only scheduled students, administered tests, and used his ingenuity in offering new ideas for developing some of the schemata norms, but also participated in assembling the final report and in making some of the drawings here presented. Miss Hermelinda M. Fogliatto and Mr. John V. Haley, together with the principal investigator, are responsible for the final presentation of this report and for having outlined the design, as well as for making concrete the several ideas dealt with in this project.

Table of Contents

Chapter Page

I. Problem On Which The Research Was Focused. 1

II. Objectives And/Or Hypotheses 3

III. Related Research 4

IV. Procedure. 7

Subjects
- a) High School Freshmen. 9
- b) College Freshmen. 9

Problems
- a) General Considerations. 10
- b) Problems Used In This Study 12
  - 1) Pre-Testing. 12
  - 2) Training Sessions. 12
  - 3) Mid-Testing Session. 16
  - 4) Post-Testing Session 16
- c) Administration of the Problems. 17

V. Analyses Of The Data And Findings. 19

- a) Matching Tests and Thought Problems. 19
- b) Grades in School 21
- c) Group Performance in the Problem 22
  - 1) Number of Cards. 22
  - 2) Correct Answers. 33
  - 3) Study of Group Performance Ellipsoids. 36
- d) Analysis of Individual Curves at the Group and Individual Levels. 41
- e) Convex Sets. 55

VI. Conclusions And Implications 70

References. 75

Appendix I. . . . . 71 pages

Appendix II . . . . 40 pages

Appendix III. . . . . 14 pages

List of Tables

Table Page

1. Experimental Design 8

2. Mean, Standard Deviation and Number of Subjects for Raven, IQ, Thought Problems, Part I and Part II, for High School and College Freshmen 10

3. Intercorrelations Between the Scores of the Tables Used . . . 20

4. Means and Standard Deviations of the Three Academic Subjects and Over-All Average of Mid-Year and Final Grades. Mean Changes from Mid-Year to Final Grades, and Differences in Changes for Experimental and Control Groups in High School Freshmen. 21

5. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each of the Twenty-Four Training Problems in Group a and b of the High School Freshmen 23

6. Mean and Standard Deviations of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each of the Twenty-Eight Training Problems in Group a and b of the College Freshmen 24

7. Mean and Standard Deviations of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Training Problems in High School and College Freshmen. 25

8. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Mid-Testing Problems of the High School Freshmen. 27

9. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Mid-Testing Problems of the College Freshmen. 27

10. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Final-Testing Problems of the High School Freshmen. 28

Table Page

11. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing and Number of Subjects Obtaining Correct Solutions for Each One of the Four Final-Testing Problems of the College Freshmen. 28

12. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Pre-Testing and Post-Testing Problems in Control and Experimental Groups of High School Freshmen 30

13. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Pre-Testing and Post-Testing Problems in Control and Experimental Groups of College Freshmen. 31

14. Total Values for the Number of Correct Solutions in the Training Sessions for High School Freshmen. 34

15. Total Values for the Number of Correct Solutions in the Training Sessions for College Freshmen. 34

16. Chi Square or Fisher's Exact Probability Test for Number of Correct Solutions in the Post-Testing Problems for Control and Experimental Groups in High School Freshmen 35

17. Chi Square or Fisher's Exact Probability Test for Number of Correct Solutions in the Post-Testing Problems for Control and Experimental Groups in College Freshmen 35

18. Ratios Between the Ellipsoids and the Parallelograms and Values of the Kolmogorov-Smirnov Test of Significance for the Pre-Testing, Mid-Testing, and Final-Testing in High School Freshmen 39

19. Ratios Between the Ellipsoids and the Parallelograms and Values of the Kolmogorov-Smirnov Test of Significance for the Pre-Testing, Mid-Testing, and Final-Testing in College Freshmen. 40

20. Four Possible Tactics for Problem 31A 43

21. Mean Length of Plateaux for Problems 26 and 41A in the Final Testing for High School and College Freshmen. 51

22. Mean Length of Plateaux for Schemata 31, 33, 35 53

23. Mean Length of Plateaux for Contents A, B, C, D 53

Table Page

24. Mean Length of Plateaux (High School Freshmen). 54

25. Mean Length of Plateaux (College Freshmen). 54

List of Figures

Figure Page

1. Schema for problems 31A, 31B, 31C and 31D 13

2. Schema for problems 31A, 31B, 31C and 31D 13

3. Schemata for problems 33A, 33B, 33C and 33D 14

4. Schemata for problems 35A, 35B, 35C and 35D 14

5. Schema for problems 37A and 37B 15

6. Graphical representation of Problem 41A 17

7. The rationale for the scoring norms for the schema presented in Figure 1, problem 31A. 43

8. Observed (Group Norms) and Weighted (Schemata Norms) Individual Performance Curves of a Special Subject for Specified Problems. 49

9. Mean Length of Plateaux of High School and College Subjects on Problems 31, 33, and 35. 52

10. Mean Length of Plateaux of High School and College Subjects on the Four Forms of the Tests. 52

11. Mean Length of Plateaux of High School Subjects on Problems 31, 33, and 35 for the Four Forms of Each Problem 56

12. Mean Length of Plateaux of College Subjects on Problems 31, 33, and 35 for the Four Forms of Each Problem 56

13. Observed Minus Expected (Group Norms) and Weighted Minus Expected (Schemata Norms) Individual Performance Curves of a Special Subject for Specified Problems. 58

14. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 26 Based on Group Norms 62

15. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 26 Based on Schemata Norms. 63

16. Limits of Performance of Loyola Control and Experimental Subjects on Problem 26 Based on Group Norms 64

17. Limits of Performance of Loyola Control and Experimental Subjects on Problem 26 Based on Schemata Norms. 65

Figure Page

18. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 41A Based on Group Norms. 66

19. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 41A Based on the Schemata Norms 67

20. Limits of Performance of Loyola Control and Experimental Subjects on Problem 41A Based on Group Norms. 68

21. Limits of Performance of Loyola Control and Experimental Subjects on Problem 41A Based on Schemata Norms 69

Chapter I

Problem On Which The Research Was Focused

The major problem of this research was the study of the effects of training in problem solving.

Previous research (Rimoldi, Devane, 1961; Rimoldi, Meyer, Meyer, Fogliatto, 1962; Rimoldi, Haley, Fogliatto, 1962) indicated that problem solving processes are highly individualized performances that can not be characterized solely by their end product. The tactics followed by a subject are important in revealing his approach to a problem. But in order to evaluate an individual's performance in a given problem it is necessary to know the characteristics and properties built in the problem by the experimenter.

In earlier attempts we tried to analyze the properties of the problems from the subjects' performances. This approach gave very useful information about each subject relative to his group and about the problem as perceived by that group. This knowledge made it possible to establish a set of specifications by which both the formal properties (schemata) and the content of the problems can be controlled by the experimenter. This turned out to be a reasonable and, we hope, acceptable way of programming problems of graded "intrinsic" difficulty. (Rimoldi, Haley, Fogliatto, Erdmann, 1962, unpublished).

Being able to better control the problems, their "difficulty" can be systematically varied by introducing new relationships in the schemata or by changing the nature of the content. Then, individual and intergroup comparisons can be made based both on the traditional statistical approach and on the formal properties and content of each problem.

One of our long standing hopes was that it should be possible, mostly when dealing with the appraisal of high talent, to evaluate subjects using norms that do not depend almost exclusively on group performances but rather on the nature of the task. A tentative solution to this problem is presented in this study. It is felt that individual differences do not need necessarily to be studied by referring to group norms that result from the combining of individual performances into a single statement. Secondly, individual performances may differ for a variety of reasons, to wit: the subject's ability to understand the logical structure of the problem, his familiarity with the content, his facility to revise modes of approach according to the nature of the problem, etc. . . .

With increased ability to control problems from the standpoint of familiarity of content and complexity of the logical structure, it was possible to study individual approaches at different specified levels. Changes in approach were studied by performance curves that indicate variations due to the complexity of the logical structure and/or the familiarity of the content. The experimental and systematic exploration of these two areas is an important part of this study.

One of our interests has been to compare individual performances when evaluated in terms of group norms and in terms of the schemata and content of the problems. Variations in performances due to differences in age, education and training can be explored. This was one of the aims of this study. Questioning the properties of the schemata in a factorial sense resulted from this approach. Though this was not an integral part of this project as originally conceived, some pilot work was performed along this line.

Specific questions raised by previous research (Rimoldi, Devane, 1961; Rimoldi, Haley, 1962a) could now be experimented upon, for instance: the effect that information presented in a negative statement may have upon problem solving ability; how this affects different individuals; the consistency of the approach followed by each subject throughout all the problems administered to him; and so forth.

In the original proposal two main types of problems were discussed. In type a the subject has to choose from a set of questions of varying degrees of relevancy to the problem those that he considers necessary to solve it. After choosing the question he obtains the corresponding information. In type b the subject generates his own questions. In this research, problems were developed for these situations.

The effects of training in problems of one type as opposed to the other and the transfer that may occur between these two were explored. The carryover from training problems to new unfamiliar problems and the performance in crucial tests, some of them repeated before and after the experiment was completed, were analyzed.

The development of appropriate scoring methods and techniques to deal with the observations and results has also been an important problem in this study. These will be detailed in the rest of the text.

Chapter II

Objectives And/Or Hypotheses

The main objectives of this study are the following:

1) To characterize problem solving processes at an individual level.

2) To study how training improves thinking processes without prescribing "best" ways of thinking or without assuming that a given problem is solvable by only one method.

Questions related to these main objectives and needed for their clarification are the following:

a) Characterization of problem solving procedures at the group level based on group norms.

b) Evaluation of the effects of training by comparison with a control group.

c) Effects that training on problems of one type have on problems of a different kind (see Procedures).

d) Effects of training in problems administered before and after the experiment proper, and their comparison with a control group.

e) Variations in processes that may be attributed to changes in schemata and in content.

f) Comparison of a, b, c, d and e above, in students at two different educational levels, i.e., high school freshmen and college freshmen.

g) Comparison of performances scored using group norms (statistical approach) with the same performance scored in terms of the properties of the problem.

Chapter III

Related Research

Max Wertheimer, in his book "Productive Thinking" (1958), suggests the possibility that present methods used in education may stifle rather than stimulate creativity. Approaches based on the theory of association, which emphasizes repetition, require a group of individuals to give a response which is associated with a particular stimulus. Other approaches attempt to analyze the type of logic involved in thinking and to develop training methods based upon this logical frame.

The teaching methods that have been developed from these two general approaches require the individual in the group to adapt himself to the method and to respond accordingly. The method described in this particular research attempts to give the individual the freedom to adapt the method to his own individual style.

This approach is a natural outgrowth of an attempt to evaluate students in terms of processes rather than final products. It was found that many different processes can lead to the same final solution or answer. When evaluation is based only on the end product, a large amount of information is lost.

One of the earlier studies attempting to evaluate individual processes was performed by Karl Duncker (1945). He gave practical and mathematical problems to subjects and asked them to "think aloud" in their attempts to solve them. The conclusions of this study were that the final solution is arrived at by successive formulations which are mediated by analyses of the conflict, material and goal.

Bloom and Broder, in their monograph "Problem Solving Processes of College Students" (1950), questioned the ability of aptitude and achievement measures to reflect the quality of the examinees' thinking. They suggested that both processes and products should compliment one another in any evaluation. In this study, they found important differences between the methods of students who passed and students who failed their comprehensive examinations. Significant improvement in grades was also observed after remedial sessions devoted to the analysis of the subjects' methods in comparison to "model" methods. In this analysis, they employed Duncker's "thinking aloud" approach.

A technique similar to the one described in this study was presented by Glaser, Damrin and Gardner (1954) and Bryan (1954). These techniques were specifically applied to the evaluation of trouble shooting in electronics. Both the administration and scoring system differ from those used in this study. These differences have been more thoroughly discussed in several papers (Rimoldi, 1955, 1960).

John and Rimoldi studied the sequential properties of complex reasoning by means of the PSI machine (John, Rimoldi, 1955; John, 1957). Although this apparatus was useful in studying certain phases of highly abstract reasoning, it was limited by the nature of problems that could be presented, the type of performance required of the subjects, and the difficulty of interpreting the results.

Buswell (1956) attempted to define common patterns in the solution of simple problems. He did this with a choice situation somewhat similar to the techniques used in this study. The results indicated more diversity than similarity in problem solving approaches.

Bruner, Goodnow and Austin (1960) examined variables that may be influential in problem processes. The conceptual formulation of this study is different than the one described in this report. In a more recent publication, Bruner discusses problem solving in relation to educational problems (1960).

Tate, Stanier, and Harootunian (1959) studied good and poor problem solvers. They found that good problem solvers are significantly better than poor in tests where quality of response, accuracy or judgement is required and without exception, the more complex the task or the more restricted the requirements, the greater their superiority. Though their study does not aim at analyzing processes in themselves, it does attempt to characterize good and poor problem solvers. A test called Thought Problems, which they prepared for their study, has been used in the present research.

Recently, D. M. Johnson (1961) found that the processes called "preparation" and "solution" could be isolated in problem solving situations and that the performance of the subjects could be predicted in terms of the training series that preceeded the crucial testing of the hypothesis.

The technique to be used in this study was devised by Rimoldi (1955). It was specifically applied to the evaluation of medical students making clinical diagnoses. In this technique, emphasis was placed on the processes leading to a diagnosis rather than on the diagnosis itself. Over a period of six years, research was performed to develop scoring methods and new forms of the test. This work is summarized in a final report entitled "The Test of Diagnostic Skills" (Rimoldi, Haley, Fogliatto, 1962). At the present time plans are being formulated to use these tests to train medical students.

This technique has been applied in a variety of areas, e.g., Rorschach interpretation (Tabor, 1959), interdisciplinary evaluation of organic pathology (Mohrbacher, 1960), appraisal of personality parameters (Gunn, 1961), decision processes in mathematical thinking (Rimoldi, Haley, Majewska, 1963, unpublished), and changes in complex mental processes with age (Rimoldi, Meyer, Meyer, Fogliatto, 1962).

The present research is a continuation of a previous study (Rimoldi, Devane, 1961), in which only high school students were used. They were trained in problems that were less clearly defined or controlled than in the present study. The training procedure was the same in both studies. Subjects were allowed complete freedom in developing and improving their own approach. Each evaluated his own procedure without any direct guidance on the part of the tester. Subjects who had been trained showed more efficiency in solving problems, greater agreement among themselves as to which questions were useful in reaching a solution, and a higher grade in mathematics. This study raised many questions related to problem solving and training, some of which will be treated in detail in this report.

Since teaching machines are designed to reinforce predefined "correct solutions", they are not comparable to the approach used in this study. The rationale underlying the use of the teaching machine is based on association, repetition and reinforcement. Other than time factors, there is no attempt to adapt the machine to a subject's individual style. We do not deny the applicability of the underlying theory itself, but prefer to avoid theorizing and instead allow the subject freedom to perform in the manner that he thinks is most suitable. In doing so, it is hoped that the possibility of the subject being forced to perform according to some theoretical model is minimized.

Chapter IV

Procedure

Table 1 presents the general experimental design. The total number of subjects used in the study was 100 of which 50 correspond to the experimental and 50 to the control group. Each group was divided into two sub-samples of subjects corresponding to 24 male college freshmen and 26 male high school freshmen.

According to the type of problems administered, each sub-sample was further divided into groups a and b. Group a includes all those subjects that were first trained in problems of type a, and later in problems of type b. In group b the order was reversed. There is a group a and a group b for both the college and high school freshmen. This distinction between groups a and b is not necessary for the control subjects.

During the pre-testing session the following tests were administered to the 100 subjects (experimental and control): Raven Progressive Matrices Test; Thought Problems, Part I, which includes the first twenty items of the test of the same name developed by Tate, Stanier and Harootunian (Tate, Stanier, and Harootunian, 1959); and problems 1, 19 and 25 as given in Appendix I.

Subjects belonging to group a were administered first the forms A, B, C and D of problems 31, 33 and 35, and forms A and B of problem 37. The high school group a was not given forms A and B of problem 37. After a mid-testing session, subjects of group a were administered forms A, B, C and D of problems 32, 34 and 36. Only the college freshmen were given forms A and B of test 38. By looking at Table 1, it should be easy to realize that the order of presentation of the problems was reversed for group b.

The mid-testing session was identical for all the experimental subjects and took place only after the first training session had been completed and before starting the second training session. The problems administered were 31C', 35A', 32E and 36E.

Problems indicated with odd numbers are of type a, while those indicated with even numbers are of type b with the exception of problems 1, 19, 25, 26 and 41A. Thus in the mid-testing session, each experimental subject was given two problems similar to the type of problems used during training and two problems of the type not yet used in training.

The final testing was identical for all the subject including controls. The instruments administered were: Thought Problems, Part II (Tate, Stanier, and Harootunian, 1959); and problems 1, 19 and 25 (that had already been administered in the pre-testing session); 31D', 35B', 32F and 36F, that correspond to problems similar to those used in the first or second training

Table 1*

Experimental Design

	Experimental Group				Control Group	
	College Freshmen		High School Freshmen		College Freshmen N = 24	High School Freshmen N = 26
	Group a N = 12	Group b N = 12	Group a N = 13	Group b N = 13		
Pre-Testing	Raven Progressive Matrices Test 1938 , Thought Problems, Part I. Problems 1, 19, 25.					
1st Training Sessions	31(A,B,C,D) 33(A,B,C,D) 35(A,B,C,D) 37(A,B)	32(A,B,C,D) 34(A,B,C,D) 36(A,B,C,D) 38(A,B)	31(A,B,C,D) 33(A,B,C,D) 35(A,B,C,D)	32(A,B,C,D) 34(A,B,C,D) 36(A,B,C,D)		Various tests were administered and meetings were held with subjects.
Mid test	31C', 35A', 32E, 36E					
2nd Training Sessions	32(A,B,C,D) 34(A,B,C,D) 36(A,B,C,D) 38(A,B)	31(A,B,C,D) 33(A,B,C,D) 35(A,B,C,D) 37(A,B)	32(A,B,C,D) 34(A,B,C,D) 36(A,B,C,D)	31(A,B,C,D) 33(A,B,C,D) 35(A,B,C,D)		
Final Testing	Thought Problems, Part II. Problems 1, 19, 25, 31D', 35B', 32F, 36F, 26, 41A.					

* N = number of subjects. The numbers and letters in the body of the Table refer to the problems described in the text.
Group a and Group b indicate whether problems of type a (cards) or type b (drawings) were used in the first training session.

sessions; and problems 26 and 41A, that were different both in content and structure from the other problems.

The control subjects (high school freshmen) were given the Rotter Incomplete Sentences Blank and the Rozensweig Picture Frustration Test, in an attempt to eliminate at least partially the Hawthorne effect.

Subjects

a) High School Freshmen.

A letter was sent to the parents of those freshmen at St. Ignatius High School in Chicago, Illinois, whose IQ in the Henmon-Nelson Test of Mental Ability was 120 or above. The nature of the research was explained and the consent of the parents was requested to let their sons participate in the study.

Seventy students were selected. Raven's Progressive Matrices Tests and Thought Problems, Part I, were administered to all of them. Twenty-six experimental-control pairs were assembled and each pair matched for IQ and score in Raven's test. Each member of the pair was randomly assigned to the experimental or control groups.

The means and standard deviations for the IQ's and Raven's Scores are given in Table 2. The paired intercorrelations were .98 for the IQ scores and .88 for the Raven Scores.

b) College Freshmen.

From a large number of college freshmen that were contacted, fifty were selected for the experiment. Two of them dropped, leaving a final total of forty-eight students.

The matching of pairs of subjects to be finally assigned to the experimental and control groups was performed using their scores in Raven's Progressive Matrices Test and Thought Problems, Part I. For this sample it was not possible to match in terms of already available IQ's, because the school records could not be easily compared. After this matching was performed, the subjects of each pair were randomly assigned to the experimental and control groups.

The mean and standard deviation for Thought Problems, Part I, and Raven's Test are given in Table 2. The paired intercorrelations were .83 for Raven's Test and .88 for Thought Problems, Part I.

In both the experimental and the control group, a few subjects dropped out in the course of the experiment. The number of subjects that took each problem is shown in Tables 5 through 13 inclusive.

Table 2

Mean, Standard Deviation and Number of Subjects
for Raven, IQ, Thought Problems, Part I and Part II,
for High School and College Freshmen

Tests		High School						College					
		Control			Experimental			Control			Experimental		
		M	σ	N of Subj	M	σ	N of Subj	M	σ	N of Subj	M	σ	N of Subj
Pre-Testing	Raven	50.27	3.93	26	49.23	4.17	26	51.75	4.73	24	52.12	4.24	24
	IQ	125.58	7.93	26	125.88	7.55	26						
	Thought Problem I	11.55	3.41	24	11.88	3.75	26	12.72	3.34	24	13.09	3.33	24
Post-Testing	Thought Problem II	13.99	3.39	24	12.64	3.98	25	15.98	1.92	21	16.66	2.40	19

Problems

a) General Considerations.

A total of 41 problems were developed of which 37 were prepared for this research. The basic rationale underlying this research is based on the assumption that complex mental processes can be described by the sequence of questions that a subject asks when solving a problem. This procedure stresses the process rather than the end product. It may be said that "the subject acts as a highly selective analyzer of stimuli and organizer of information. At every discrete experimental stage the nature of the problem changes. Accumulation of relevant knowledge and decrement of ignorance can be specifically related to every stage in the process, provided this stage can be identified experimentally." (Rimoldi, Haley, Fogliatto, Erdmann, 1962, unpublished).

Two general types of problems were constructed. In the first type (a) the subject is presented with a problem and a set of cards containing questions that he may ask in order to reach a solution. The subject is free to choose any card in any order he wishes. The corresponding answer to each question is written on the reverse side of the card. When the subject thinks that he has gained sufficient information to solve the problem correctly, he stops selecting cards and gives his solution. The experimenter records the questions in the order in which they were asked. These problems have no time limit.

In the second type of problem (b) a drawing or figure is presented to the subject who is asked to identify some part of the figure. Here the subject is allowed to generate his own questions and the experimenter provides the answer. When the subject feels that he can accurately identify the prescribed area of the drawing he stops asking questions and gives his answer. Since each part of the figure presented to the subject is identified by a number or a letter, it is an easy matter for the experimenter to know the sequence of questions asked. These problems have no time limit.

Developing techniques to evaluate these results has been a very difficult problem. Many approaches have been tried with varying degrees of success (Rimoldi, Grib, 1960a, 1960b; Rimoldi, Devane, Haley, 1961). A method was finally developed which took into consideration both frequency and order of selection (Rimoldi, Haley, 1962a, 1962b). This approach was based upon norms that depended at least partially on group performance.

As stated above a problem of crucial importance was to develop norms that were dependent upon the nature of the problem itself rather than the performance of a particular group. It was in trying to solve this problem that ways were devised for preparing instruments in such a way that their content and structure could be controlled.

The word "structure" as used in this report refers to the formal properties or schema of the problem expressed in terms of a basic set of logical relationships. "These 'schemata' are the logical frames on which various types of objects may be superimposed. By changing the formal properties of problems, various levels of complexity can be defined. Each one of these levels is thus objectively characterized, so that different investigators will have the same set of references when using a given schema." (Rimoldi, Haley, Fogliatto, Erdmann, 1962, unpublished). For this study a graded set of schemata was prepared.

"The objects (content) used to realize the problems may consist of words, numbers, letters, negative statements, drawings, etc., representing concrete or abstract propositions. The same logical frame can be presented with content that refers to every day familiar situations or in a problem dealing with symbols, familiar or unfamiliar to the subject, etc. Since every schema can be presented using different classes of content and since the same type of content can be used in different schema, an experimental program for the study of the importance of these components in the thinking process can be developed.

"The difficulty of each problem becomes an 'intrinsic' property of the problem and is defined in terms of the relationships used in the corresponding schema. This difficulty can be experimentally operated on by changing the type and number of relations used.

"The objects used to realize the problem can be drawn from a variety of sources. The familiarity with these sources may be important in the solution of the problem. Thus the same schema with different content may bring about different performances in the same subject." (Rimoldi, Haley, Fogliatto, Erdmann, 1962, unpublished).

The logical analysis of the "schemata" provides a set of norms that may be considered to be experimentally independent to the norms developed from group performance in the problems. The first will refer to "intrinsic" difficulty, while the second, corresponds to the "extrinsic" difficulty of the problems that reflects intrinsic difficulty, familiarity with content and a host of other variables difficult to isolate, i.e., individual differences, sampling errors, etc.

b) Problems Used In This Study

1) Pre-Testing

Problems 1, 19 and 25 were used in the pre-testing session. They were obtained from a previous study (Rimoldi, Devane, 1961) and were not built following the specifications described above. (see Appendix I).

These problems serve a twofold purpose: to acquaint subjects with the technique to be used in the experiment proper; and to compare subjects' abilities to perform prior to training sessions.

2) Training Sessions

Problems 31 to 36 inclusive had four forms each (A, B, C, D) while problems 37 and 38 had only two forms (A and B). These problems were used in the training sessions as shown in Table 1. Type a problems will be described first, i. e., 31, 33, 35 and 37 in all their forms. The designation for problems of type a as A, B, C or D hinges upon the content used, while the numbers 31, 33, 35 and 37 refers to the schemata used.

Type a Problems (31, 33, 35, 37).

Form A of the problems involves concrete every day situations, i.e., school, employment, etc., as can be seen in Appendix I. The content of form B of the problems is letters that represent symbolically non-specified concrete objects. In problems of form C the questions are presented using negative statements, i.e., . . ."Are there more not-D's than not-A's among the F's?". In forms A, B and C the solution of the problem requires the use of numbers.

D problems are similar to B problems in as far as letters stand for non-specified concrete objects. The solution here requires the use of letters instead of numbers so the idea of quantity is not necessary for the solution, i.e.,. . ."How many L's are M's? Answer: Y + F = X". These represent the highest degree of abstraction in our set of problems. The schema for problems 31A, 31B, 31C, and 31D may be represented as a tree though other graphical representations may be used. In Figure 1 their basic logical structure is depicted.

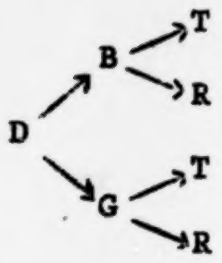


Figure 1

In Figure 1, which corresponds to problem 31A, the letter D represents the high school dance committee, B stands for boys and G for girls in the committee, T and R for individuals who sell tickets or take care of refreshments respectively. (See Appendix I).

If we call the dance committee C, and boys B, girls G, tickets T and refreshments R, we obtain problem 31B. By calling D not-X, and by calling B not-A, G not-D, T not-P and R not-S, we obtain problem 31C. Problem 31D will be similar to problem 31B, though different letters will be used, but, as stated above, the answers to the questions are in letters rather than numbers.

It should be noticed that Figure 1 could be also represented by a schema in which the first two branches lead to T and R, and the final branches to B and G as shown in Figure 2.

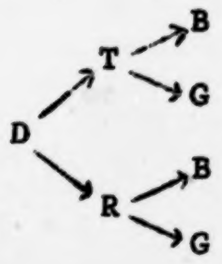


Figure 2

The remaining problems will be more summarily described. The interpretation of them is along the lines used to describe problem 31.

In Figure 3, representations of problems 33A, 33B, 33C and 33D, are given.

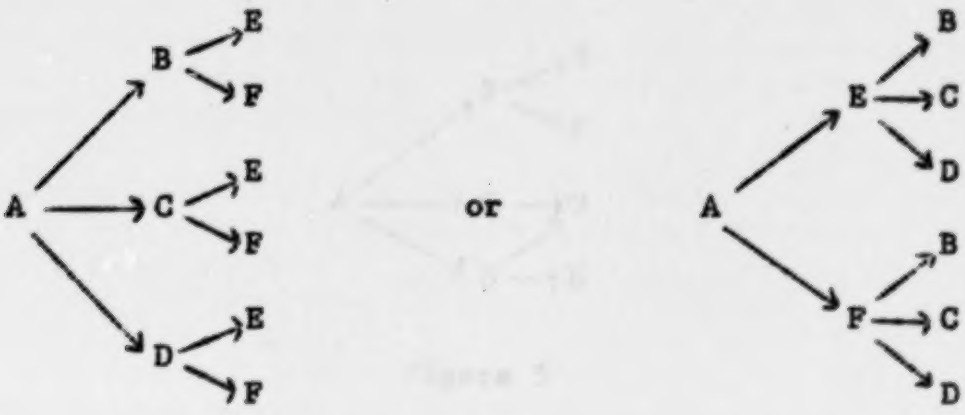

A
B
C
D
E
F
E
F
E
F
or
A
E
F
B
C
D
B
C
D

Figure 3

By comparison with all the problems numbered 31, those listed as 33 have a tree with both a dichotomy and trichotomy.

In Figure 4, which gives the schemata of problems 35A, 35B, 35C and 35D, two trichotomies are involved.

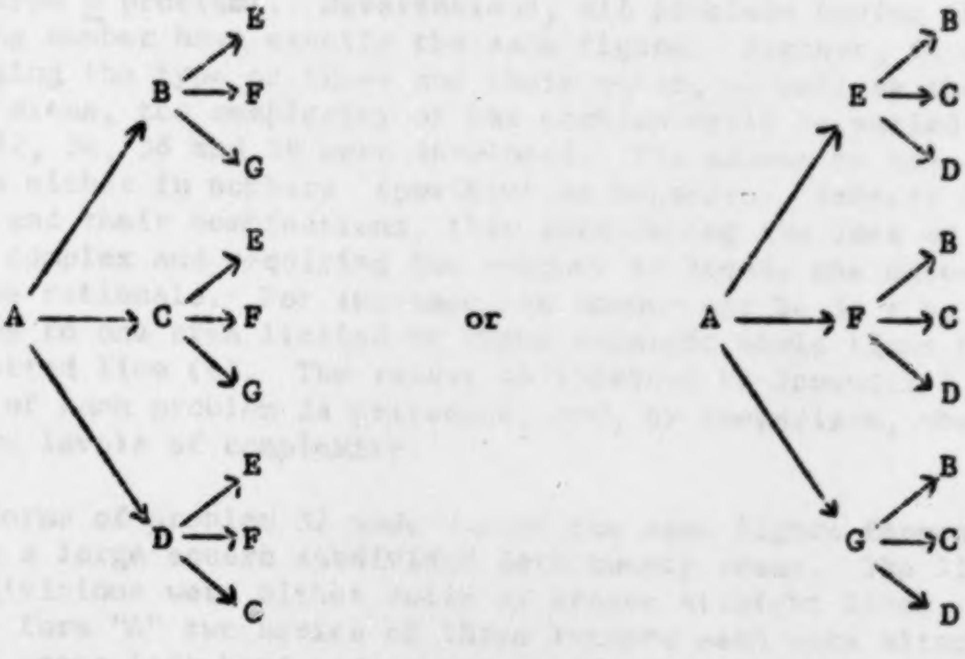

A
B
C
D
E
F
G
E
F
G
E
F
C
or
A
E
F
G
B
C
D
B
C
D
B
C
D

Figure 4

A convergence of branches is introduced into the trees that represent problem 37A and 37B. The schema for these problems is presented in Figure 5.

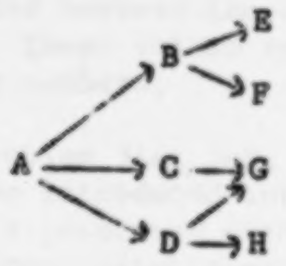


Figure 5

Inspection of Figures 1 to 5 indicates how the complexity of the schemata may be controlled. This may refer to number of branches, different combinations of subdivisions of the branches, their convergence or divergence, etc. These components may be varied at will, within restrictions implied mostly by practical limitations. Schemata of graded "intrinsic" difficulty may be built and different "contents" used to realize them.

Type b Problems

No general statement can be made about the content of Forms A, B, C, and D of type b problems. Nevertheless, all problems having the same identifying number have exactly the same figure. Further, it was assumed that changing the type of lines and their color, as well as changing the number of areas, the complexity of the problem could be varied. Thus, problems 32, 34, 36 and 38 were developed. The answer to the questions were given either in numbers (positive or negative), letters (positive or negative) and their combinations, thus introducing the idea of series, from simple to complex and requiring the subject to decode the answers in order to find the rationale. For instance, an answer may be 3a + b, because this corresponds to one area limited by three straight whole lines (a) and a colored dotted line (b). The reader is referred to Appendix I where the rationale of each problem is presented, and, by comparison, observe the progressive levels of complexity.

All forms of problem 32 made use of the same figure (Appendix I). This figure was a large square subdivided into twenty areas. The lines used for these subdivisions were either solid or broken straight lines, or curved lines. In form "A" two series of three letters each were alternated starting at the upper left hand corner and moving from left to right, top to bottom.

In form B a letter series and a number series were used. Although letters and numbers were alternated in groups of three, each series was continued from the last step, for instance, . . . abc 123 def 456. Form C is similar to form B, however, only numbers are employed, one set positive, the other negative. Numbers were also used in form D. This form of the test was further complicated because the values added to each number in the series were not constant. These values were also in series and included both positive and negative numbers.

The figure used in problem 34A, 34B, 34C and 34D was subdivided into 36 areas. Colors were also introduced into the problems. The rationale underlying each solution is presented with the problems in Appendix I. Again each form was made progressively more complex.

The four forms of problem 36 employed a figure subdivided into 20 areas. Colors were used as well as solid and broken, curved and straight lines. The solution to the four forms of the problem was dependent upon associating positive or negative letters with different varieties and colors of lines. (Appendix I). In problems 38A and B the concept of symmetry was implied.

The rationale used to build all the problems, as given in Appendix I was determined exclusively by the experimenters who developed the problems. We do not claim, though, that this rationale will be employed by all the subjects. As a matter of fact, some approaches, of which we were unaware, were used with success by the subjects in solving some of the problems.

3) Mid-Testing Session

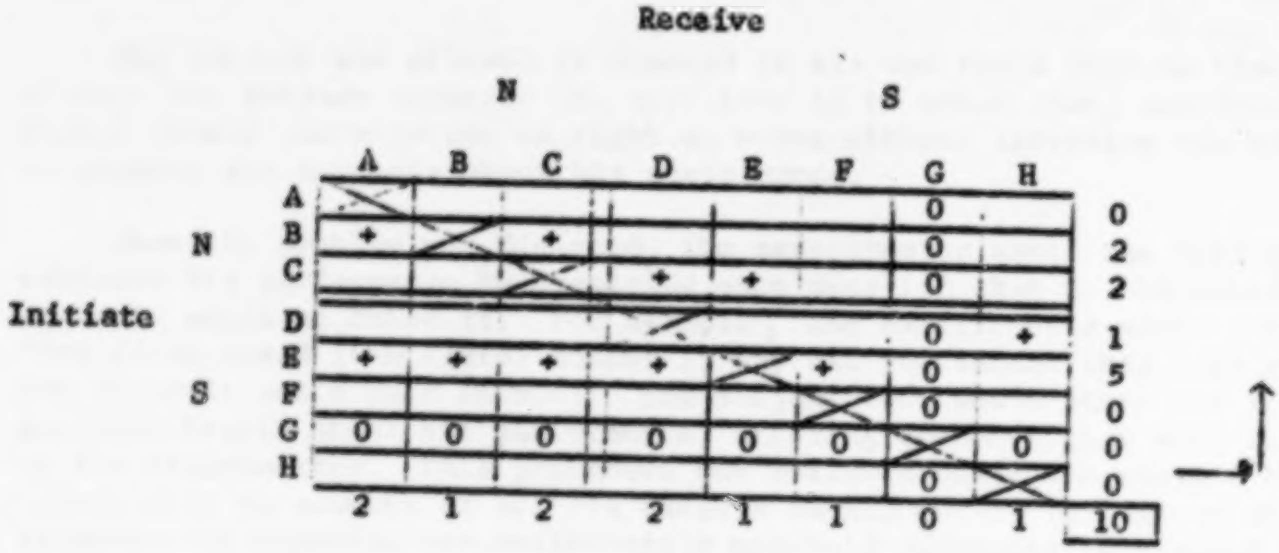
Problems 31C', 35A', 32E and 36E were administered in the mid-testing session. The first two problems are exactly the same as 31C and 35A as far as schemata and type of content are concerned. Nevertheless, as it can be seen in Appendix I, the specific content is new. The same thing applies to problems 32E and 36E.

4) Post-Testing Session

Problems 1, 19 and 25 are exactly the same as those in the pre-testing session. Problems 31D', 35B', 32F and 36F follow the rationale explained before for problems of type a and b. The problems themselves can be found in Appendix I.

Problem 26 was a new problem, selected among those that had shown better promise in a previous investigation (Rimoldi, Devane, 1961). It could be described in terms of trees but when it was developed we proceeded by a process of elimination of pertinent information that converges in the final answer.

Problem 41A refers to a communication network, the graphical representation of which is given in the matrix in Figure 6.



Initiate		Receive								
		N			S					
		A	B	C	D	E	F	G	H	
N	A	X						0		0
	B	+	X	+				0		2
	C			X	+	+		0		2
S	D				X			0	+	1
	E	+	+	+	+	X	+	0		5
	F						X	0		0
	G	0	0	0	0	0	0	X	0	0
	H							0	X	0
		2	1	2	2	1	1	0	1	10

Figure 6

Assume that N and S are two buildings in a factory. Building N has three offices A, B and C, and building S has five offices, D, E, F, G and H. The arrows on the right hand side of the figure indicate the direction in which communications are effected between offices. Thus, from office B it is possible to initiate a call to offices A and C. (Notice that from A no call can be initiated). The row and column of 0's indicate that office G can neither receive nor initiate a call.

The column marginals show from how many offices each office may receive calls, i.e., . . .office A can be called from two offices, B and E. The row marginals indicate how many offices can be contacted from a given office, i.e., from E, five offices can be contacted.

This schema can be further complicated by increasing the number of cells, by increasing the main subdivisions in the table, by using a double entry system and so forth. As indicated in Appendix I, Figure 6 was used in this research.

c) Administration Of The Problem

Problems 1, 19 and 25 as well as Raven's Progressive Matrices and Thought Problems, Part I, were administered in group form. No time limit was prescribed for problems 1, 19 and 25. The remaining tests were administered according to the specifications in the test manuals. Problems 1, 19 and 25 were not reviewed with the experimenter as it was done in the training sessions.

The training session problems were administered individually in the order indicated in Table 1. Each testing session lasted approximately for one hour.

The subject was allowed to proceed at his own speed with no time limit given. The subject recorded the questions as he asked them, and the experimenter scored the solution as right or wrong without informing the subject or passing any comments about his performance.

When the problem was finished, the experimenter asked the subject to evaluate his performance by examining each question that he had asked in the order in which he asked it. For example , the experimenter would say: "You first asked item (card) number 7; why did you select this card and do you think it was a good choice?" The subject then would state his reasons and talk freely about his performance, offering comments that were recorded by the experimenter. This procedure was followed until the subject had an opportunity to comment on all the choices he had made. Throughout these sessions the experimenter deliberately withheld judgments that could be interpreted as his evaluation of the subject's performance or that could be construed as suggestions for "better" approaches. This technique was used in all the problems administered during the training sessions.

The problems administered during the mid-testing session were also given individually. No review session was performed here.

The post-testing problems were administered in group form. No review session took place. Thought Problems, Part II, were administered according to the test instructions.

The control group did not have training sessions or mid-testing session. The pre- and post-testing sessions were identical to those for the experimental group.

The control subjects were administered individually the Rotter Incomplete Sentences Blanks and the Rozenweig Picture Frustration Test, in an attempt to control for the so-called "Hawthorne" effect. Because of time limitations and difficulties in scheduling the control college freshmen, only a few of them were administered these tests.

Chapter V

Analyses Of The Data And Findings

a) Matching Tests and Thought Problems

We shall analyze first the results obtained in the pre- and post-testing sessions for both the experimental and control groups of high school and college freshmen.

Table 2 presents the means, standard deviations and number of subjects on which these statistics are based. As expected, because of the individual matching, there are no differences, at an acceptable statistical level, between the control and experimental group in the means of the Raven's test (college and high school freshmen), in the Henmon-Nelson IQ Test (high school freshmen), or in Thought Problems, Part I (college freshmen).

The comparison of the results obtained between control and experimental groups in Thought Problems, Part I, in both college and high school freshmen show no significant differences, either for means or variances. The same holds for Thought Problems, Part II, in the post-testing sessions.

The matched pair correlations are the following:

High School, Raven Test	.88
High School, Henmon-Nelson IQ	.98
College, Raven Test	.83
College, Thought Problems, Part I	.88

These values thus far reported are an indication that the matching was satisfactorily performed.

It is interesting to note that the matched pair correlation for Thought Problems, Part I, in high school freshmen is .44. This test was not used for matching purposes in this group.

The intercorrelations of the scores between the tests used are as shown in Table 3. All the intercorrelations thus far reported are all significant at the .01 level, with the exception of the intercorrelations between Thought Problems, Part I, and the Henmon-Nelson IQ in high school freshmen (experimental group) and Raven Versus Thought Problems, Part I, in college freshmen (experimental group) that are significant at the .05 level.

Comparing the means corresponding to Thought Problems, Part I and Part II, administered respectively in the pre- and post-testing sessions it is found that: the increase in scores is significant for both control and experimental college freshmen, at the .001 level; but only for the control group among high school freshmen (.01 level) and not for the experimental group.

Table 3

Intercorrelations Between the Scores of the Tests Used

	High School		College	
	Experimental	Control	Experimental	Control
Raven's vs Thought Problems, Part I	.56*	.64*	.49	.72
Raven's vs IQ (Henmon-Nelson)	.67	.56		
IQ (Henmon-Nelson) vs Thought Problems, Part I	.43*	.55*		

* The values with an asterisk refer to intercorrelations in which one of the tests was not used for matching purposes.

The next analysis refers to the comparisons between high school and college freshmen for the control and experimental groups. The results indicate that there is no significant difference in Thought Problems, Part I (pre-testing) between high school and college controls or high school and college experimentals. That is, statistically considered, the high school and college groups are the same at the beginning of the experiment.

At the end of the experiment, these same comparisons show a significant increase in scores (.01 level), in favor of the college group.

The study of the variances shows that for the high school group, neither the control nor the experimental differs at any statistically acceptable level when comparing performances in Thought Problems, Part I and II. Nevertheless, in the college group the change is significant at the .05 level for the experimental and at the .01 level for the control group. The test here used is the t statistics for correlated variances.

Summarizing, in the college group the increase in scores is more definite than in the high school sample and is accompanied by a corresponding increase in homogeneity.

The fact that the differences between the means of high school and college freshmen are significant for Thought Problems, Part II, may be attributed to the effects of a variety of sources, among them, age, educational level, etc. We hope to clarify further this point in the following text.

b) Grades in School

In Table 4 the means and standard deviations of grades in three subjects (English, Latin and Algebra) as well as an average grade are given for the control and experimental high school freshmen. The differences in grades obtained at the mid-year and at the end of the year are given. These differences are not significant. However, when these differences are compared, that is, when the changes for the experimental and the control group are subtracted it is found that the differences in changes are significant at the .05 level for the average grade, in favor of the experimental group. That is, the gain in the experimental group is greater than in the control group.

Table 4

Means and Standard Deviations of the Three Academic Subjects
and Over-All Average of Mid-Year and Final Grades.
Mean Changes from Mid-Year to Final Grades,
and Differences in Changes
for Experimental and Control Groups
in High School Freshmen.

	Experimental					Control					Difference in Mean Change
	Mid-Year		Final		Changes in M	Mid-Year		Final		Changes in M	
	M	σ	M	σ		M	σ	M	σ		
English	84.37	6.35	84.12	7.46	- .25	86.00	6.87	84.92	7.42	-1.08	.83
Latin	82.61	7.45	83.87	5.64	1.26	84.96	8.23	85.09	7.53	.13	1.13
Algebra	81.17	7.51	81.83	6.66	.65	83.91	6.90	84.78	6.52	.87	- .22
[1] Average	82.33	5.92	83.69	6.24	1.36	84.71	6.02	84.77	5.64	.06	1.30*

*$p \leq .05$ (t test)

[1] The average was based on five academic subjects beyond those listed here.

c) Group Performance in the Problem

1) Number of Cards

As described elsewhere, this score consists essentially in the number of cards chosen by each subject in order to solve the problem. It is assumed that number of cards is partially a function of efficiency in interpreting information and asking questions. Other factors may also enter in this type of score (Rimoldi, Devane, Grib, 1958).

Tables 5 and 6 show the means and standard deviations of number of cards for each problem administered during the training sessions for high school and college freshmen respectively.

Table 7 gives the same statistics without differentiating the training sessions in which the problems were administered.

According to our design, problems of type *a* were administered first to group *a*, and problems of type *b* to group *b*, so that in the second training session, subjects of group *a* were given problems of type *b*, and subjects of group *b* were given problems of type *a*. Thus, if there is transfer from one type of problem to the other type, a change in the corresponding statistics should be observable.

By comparing mean number of cards in college freshmen for type *a* problems in the first session and type *a* problems in the second session (when the subjects had already been trained in problems of type *b*) it is found that, in 11 out of 14 problems, there is a decrement in the number of cards used in group *b*. This over-all change is significant (one tailed binomial test) at the .029 level. When studying the group that received, in the first training sessions, problems of type *b* with those who received problems of type *b* in the second session, that is after having had training in problems of type *a*, it is found that in 11 out of 14 problems there is a decrement in the number of cards used. Again, this is significant at the .029 level.

The previous data refers to college freshmen (Table 6). For high school freshmen, we found that in problems of type *a*, 9 out of 10 *increase* the mean number of cards (two problems have exactly the same mean) after having had training with problems of type *b*. For problems of type *b*, 10 out of 12 decrease the mean number of questions used after having had training with problems of type *a*. This last set of values is significant at the .019 level (binomial test). The corresponding data is presented in Table 5.

In summary, studying the total groups of college and high school freshmen, it is found that the number of questions asked in the second training session in problems of type *a*, when compared with the number of questions asked in the first training session in the same type of problems, is less in 12 out of 24 cases. When performing the same operation for the type *b* problems it is found that the number of questions is less in 21 out of 26 cases.

Table 5

Mean and Standard Deviation of Number of Questions,
Number of Subjects Completing, and Number of Subjects Obtaining
Correct Solutions for Each of the Twenty-Four Training Problems
in Group a and b of the High School Freshmen

Problems	Group a				Problems	Group b			
Type a	M	σ	N of Subjects	Correct Solution	Type b	M	σ	N of Subjects	Correct Solution
31 A	3.77	..97	13	12	32 A	15.00	2.96	13	11
B	6.38	1.86	13	7	B	14.92	3.81	13	12
C	5.46	2.21	13	6	C	14.85	3.96	13	9
D	5.92	1.44	13	6	D	15.38	2.98	13	10
33 A	6.54	1.99	13	10	34 A	20.15	8.78	13	11
B	7.23	1.89	13	9	B	18.62	5.33	13	12
C	6.54	2.10	13	10	C	14.69	7.24	13	12
D	6.08	1.86	13	12	D	17.54	4.05	13	8
35 A	5.61	1.39	13	11	36 A	16.15	4.00	13	3
B	7.31	2.95	13	8	B	12.85	5.43	13	9
C	5.83	1.46	12	7	C	12.00	5.96	13	3
D	6.23	1.97	13	9	D	11.42	5.68	12	5
Type b					Type a				
32 A	10.69	3.07	13	13	31 A	3.77	.80	13	12
B	10.83	3.26	12	11	B	5.92	1.86	13	6
C	11.75	3.65	12	11	C	6.69	2.58	13	3
D	14.33	3.64	12	2	D	5.92	1.69	13	5
34 A	15.62	6.37	13	13	33 A	8.23	2.01	13	8
B	19.38	5.97	13	12	B	9.46	2.06	13	10
C	15.31	5.86	13	11	C	7.69	1.77	13	10
D	16.62	6.17	13	11	D	7.54	2.37	13	12
36 A	14.00	4.21	13	2	35 A	6.00	1.68	12	10
B	11.46	4.53	13	10	B	8.69	3.05	13	7
C	11.77	5.44	13	5	C	8.08	3.00	13	11
D	10.00	5.97	13	5	D	7.15	2.96	13	11

Table 6

Mean and Standard Deviations of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each of the Twenty-Eight Training Problems in Group a and b of the College Freshmen

Problems	Group a				Problems	Group b			
Type a	M	σ	N of Subjects	Correct Solution	Type b	M	σ	N of Subjects	Correct Solution
31 A	3.75	.72	12	12	32 A	10.92	2.36	12	8
B	5.25	1.92	12	6	B	10.33	3.79	12	12
C	4.75	.92	12	7	C	11.00	2.94	12	12
D	4.50	1.04	12	6	D	13.42	3.90	12	8
33 A	5.75	1.83	12	12	34 A	13.73	5.96	11	10
B	6.75	1.36	12	11	B	15.00	3.95	11	11
C	6.83	1.77	12	11	C	15.00	3.44	11	10
D	5.67	1.80	12	11	D	14.18	4.26	11	11
35 A	6.00	1.73	12	12	36 A	15.45	4.52	11	3
B	6.50	2.25	12	9	B	11.09	5.47	11	7
C	6.25	1.64	12	11	C	8.36	5.88	11	9
D	6.17	1.52	12	12	D	4.00	1.54	11	11
37 A	6.92	1.66	12	7	38 A	24.82	4.95	11	11
B	7.75	1.01	12	12	B	25.09	7.01	11	9
Type b					Type a				
32 A	11.60	3.10	10	10	31 A	3.82	.83	11	10
B	9.70	2.19	10	9	B	4.91	2.06	11	6
C	11.70	2.45	10	9	C	3.36	.64	11	7
D	11.70	3.52	10	8	D	4.09	.79	11	10
34 A	13.40	4.25	10	8	33 A	4.64	.77	11	9
B	11.90	2.77	10	10	B	7.00	2.17	11	9
C	10.90	2.81	10	9	C	5.54	1.78	11	11
D	12.50	2.33	10	10	D	5.64	1.97	11	10
36 A	13.10	5.39	10	5	35 A	5.36	.88	11	11
B	9.10	4.61	10	5	B	6.09	1.78	11	9
C	5.44	1.34	9	8	C	5.00	0	11	11
D	4.44	1.16	9	9	D	5.00	.43	11	11
38 A	17.00	6.74	8	7	37 A	7.09	1.73	11	7
B	19.62	7.94	8	7	B	7.00	1.65	11	9

Table 7

Mean and Standard Deviations of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Training Problems in High School and College Freshmen

Problems	High School				College			
Type a	M	σ	N of Subjects	Correct Solution	M	σ	N of Subjects	Correct Solution
31 A	3.77	.89	26	24	3.78	.78	23	22
B	6.15	1.88	26	13	5.09	2.00	23	12
C	6.07	2.48	26	9	4.09	1.06	23	14
D	5.92	1.57	26	11	4.30	.95	23	16
33 A	7.38	2.17	26	18	5.22	1.53	23	21
B	8.34	2.27	26	19	6.87	1.80	23	20
C	7.11	2.03	26	20	6.22	1.89	23	22
D	6.81	2.25	26	24	5.65	1.88	23	21
35 A	5.80	1.55	25	21	5.70	1.43	23	23
B	8.00	3.08	26	15	6.30	2.05	23	18
C	7.00	2.64	25	18	5.65	1.34	23	22
D	6.70	2.55	26	20	5.61	1.28	23	23
37 A					7.00	1.69	23	14
B					7.39	1.41	23	21
Type b								
32 A	12.85	3.71	26	24	11.23	2.75	22	18
B	12.96	4.10	25	23	10.04	3.18	22	21
C	13.36	4.12	25	18	11.32	2.75	22	21
D	14.88	3.35	25	12	12.64	3.83	22	16
34 A	17.88	8.00	26	24	13.57	5.22	21	18
B	19.00	5.67	26	24	13.52	3.77	21	21
C	15.00	6.59	26	23	13.05	3.76	21	19
D	17.08	5.72	26	19	14.05	2.01	21	21
36 A	15.08	4.24	26	5	14.33	5.09	21	8
B	12.15	5.05	26	19	10.14	5.17	21	12
C	11.88	5.71	26	8	7.05	4.68	20	17
D	10.68	5.88	25	10	4.20	1.40	20	20
38 A					21.53	6.95	19	18
B					22.79	7.89	19	16

Since, in the first case, subjects had been trained in type b problems and, in the second case, they had been trained in type a problems, it seems reasonable to conclude that training in type a problems increases the efficiency of solving type b problems more than type b problems increase the efficiency of solving type a problems. In fact, the high school freshmen that had been trained with problems of type b, instead of decreasing, asked more questions in 9 out of 10 cases (two cases being the same). This is difficult to explain unless the idea of a negative transfer is offered. At first examination this does not seem too logical. Other reasons may also be brought forth, such as age of the subjects, testing conditions, interests of the member of the sample, educational level, etc.

The means and standard deviations of the number of cards used in the mid-testing session are presented in Table 8.

Subjects who had been trained in problems of type a selected fewer cards in problems of type a than subjects who had been trained with type b problems. This holds both for the high school and college freshmen. In all cases, with the exception of problem 35A' in college freshmen, the significance level of the difference is .05 or better. This can be seen by inspecting the t values in Tables 8 and 9.

Similar results are obtained comparing the number of questions used by subjects that have been trained in problems of type b, with those that have been trained in problems of type a, when the mid-test problem is of type b. The results show that the differences are significant for both high school and college freshmen at the .05 level or better in problem 36B (see t values in Tables 8 and 9). This does not occur for problem 32B, where the differences are not significant.

Similarly, the inspection of the standard deviation corresponding to number of questions asked indicates that when the problem used for mid-testing is of the same type as the problems used in the training sessions, the variance is smaller, with the exception of problem 36B in high school students.

It may then be suspected that training in one type of problem is highly effective in problems of the same type in economy of questions asked and in reducing individual variations in number of questions asked, with the exceptions listed in the text.

The final testing session included both control and experimental groups in problems 1, 19 and 25 (that were the same in the pre- and post-testing sessions), problems 26 and 41A that were entirely new, and problems 31D', 35B', 32F and 36F.

The experimental group performance in the last four tests mentioned will be analyzed first. By looking at Tables 10 and 11, it is clear that the differences between the mean number of cards used is not significant.

Table 8

Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Mid-Testing Problems of the High School Freshmen

Problems	Group a				Group b				
Type a	M	σ	N of Subjects	Correct Solution	M	σ	N of Subjects	Correct Solution	"t" value
31 C'	5.31	1.54	13	7	7.62	2.34	13	2	2.86^{xx}
35 A'	6.00	.96	13	9	9.23	2.26	13	10	4.56^{xxx}
Type b									
32 E	12.69	3.73	13	8	12.85	3.74	13	10	- .10
36 E	14.62	3.97	13	1	11.00	5.88	13	6	1.77^{x}

^{x} $p < .05$
^{xx} $p < .01$
^{xxx} $p < .001$

Table 9

Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Mid-Testing Problems of the College Freshmen

Problems	Group a				Group b				
Type a	M	σ	N of Subjects	Correct Solution	M	σ	N of Subjects	Correct Solution	"t" value
31 C'	4.50	1.04	12	7	5.91	2.19	11	3	1.96^{x}
35 A'	6.25	1.36	12	12	7.54	2.61	11	9	1.47
Type b									
32 E	9.75	4.06	12	3	9.63	2.19	11	6	.09
36 E	11.58	4.05	12	2	5.09	2.57	11	11	4.63^{xxx}

^{x} $p < .05$
^{xxx} $p < .001$

Table 10

Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Final-Testing Problems of the High School Freshmen

Problems	Group a				Group b				
Type a	M	σ	N of Subjects	Correct Solution	M	σ	N of Subjects	Correct Solution	"t" value
31 D'	4.23	1.67	13	11	4.85	1.51	13	11	.95
35 B'	5.92	1.64	13	7	6.75	2.42	12	8	.97
Type b									
32 F	15.15	4.38	13	7	14.69	3.07	13	7	.31
36 F	9.46	5.54	13	9	8.85	5.75	13	11	.26

Table 11

Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Four Final-Testing Problems of the College Freshmen

Problems	Group a				Group b				
Type a	M	σ	N of Subjects	Correct Solution	M	σ	N of Subjects	Correct Solution	"t" value
31 D'	3.88	.78	8	8	3.82	.86	11	10	.14
35 B'	5.88	1.69	8	7	5.91	1.24	11	11	.05
Type b									
32 F	14.12	3.33	8	6	14.00	2.73	11	7	.08
36 F	5.38	2.34	8	8	5.09	4.23	11	10	.18

The fact that at this post-testing session, none of the pairs of means show significant differences is highly remarkable when interpreted in terms of the findings given in Tables 8 and 9, that correspond to the mid-test session. As it will be remembered, the general conclusion was that training in one type of problems is effective in performance in the same type of problem. From the comparisons indicated in Tables 10 and 11, we can assume that this conclusion is further strengthened, in the sense that at the final stage of the experiment all the subjects had received training in both types of problems. Therefore, if the previous conclusions were acceptable, the results of the final testing should indicate no differences. Also, at this stage in the experiment, it matters little whether problems of type a or type b were given first, that is, within the experimental limitations imposed by our design, order of administration of problems seems to be irrelevant, since none of the comparisons is statistically significant at an acceptable level.

It is interesting that when the means and standard deviations of corresponding tests in Tables 10 and 11 are compared for both high school and college freshmen, the general trend is that high school freshmen use more questions than college freshmen, and also have higher variances. The difference between means is significant for test 36F (one tail test) at the .05 level.

The variances for the high school freshmen are higher than those for the college freshmen, reaching the .05 level of significance for problem 31D'. For problems 35B' and 36F, the same thing happens for groups b and a respectively.

In summary, the trend indicates that college freshmen ask less questions and are more homogeneous than high school students. This may be an interesting hint deserving further exploration as to the advantages and possibilities of training in problem solving at different ages and educational levels.

In Tables 12 and 13 we present the same type of data for both control and experimental groups in Tests 1, 19 and 25 at the pre- and post-testing sessions, as well as for problems 31D', 35B', 32F, 36F, 26 and 41A.

Concerning problems that were administered both at the pre- and post-testing session to both experimental and control groups, it is found that there is no difference for the high school freshmen (Table 12) in the mean number of cards chosen in problems 1, 19 and 25. For the college students in problem 19, pre-testing, the control group selects more cards than the experimental group. The difference is significant at the .05 level. Comparing now the number of cards chosen by the same subjects in problems 1, 19 and 25 in both the pre-testing and post-testing sessions, it is found that for high school students the only differences that are significant correspond to problem 1 at the .001 level in the control group and the .05 level in the experimental group, the mean number of cards in both cases being higher in the pre-testing session. In the college freshmen, comparing pre- and post-testing sessions, it is found that in the control group there is a decrement

Table 12

Mean and Standard Deviation of Number of Questions Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Pre-Testing and Post-Testing Problems in Control and Experimental Groups of High School Freshmen

Problems		Control				Experimental				"t" value[1]
		M	σ	N	Correct Solution	M	σ	N	Correct Solution	
Pre-Testing	1	4.04	1.60	26	24	4.08	1.96	26	25	.08
	19	8.08	2.97	26	15	8.28	2.52	25	16	.16
	25	7.31	3.75	26	19	6.81	2.54	26	22	.51
Post-Testing	1	2.96	.68	24	24	3.27	1.29	26	25	1.23
	19	8.21	2.61	24	18	8.88	1.99	26	22	1.11
	25	6.29	2.05	24	22	6.77	2.15	26	23	.70
Type a	31D'	5.92	2.14	24	5	4.54	1.62	26	22	2.52**
	35B'	7.88	2.18	24	9	6.32	2.09	25	15	2.63**
Type b	32F	15.46	2.78	24	12	14.92	3.79	26	14	.72
	36F	13.75	4.26	24	2	9.15	5.65	26	20	3.71***
New Problems	26	7.96	2.19	24	9	7.35	2.34	26	14	.83
	41A	7.04	2.30	24	12	5.65	1.80	25	19	2.20*

*$p < .05$
**$p < .01$
***$p < .001$

[1] The "t" values were computed for correlated means (matched pairs).

Table 13

Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solutions for Each One of the Pre-Testing and Post-Testing Problems in Control and Experimental Groups of College Freshmen

Problems		Control				Experimental				"t" value[1]
		M	σ	N	Correct Solution	M	σ	N	Correct Solution	
Pre-Testing	1	3.087	.8295	23	22	3.167	1.213	24	23	- .50
	19	9.000	2.485	23	20	7.458	1.756	24	17	2.52*
	25	6.348	2.277	23	18	5.333	1.675	24	23	1.76
Post-Testing	1	2.739	.6056	23	21	2.474	.5955	19	18	1.55
	19	7.870	1.7271	23	17	7.526	1.2296	19	16	.0
	25	5.217	1.1018	23	19	4.737	1.0178	19	16	.39
Type a	31D'	5.565	2.102	23	7	3.842	.8120	19	18	3.24**
	35B'	7.478	2.282	23	9	5.895	1.447	19	18	2.20*
Type b	32F	14.478	2.652	23	10	14.053	2.9995	19	13	1.12
	36F	14.478	3.335	23	3	5.211	3.562	19	18	7.10***
New Problems	26	7.783	1.817	23	9	6.158	1.694	19	11	3.32**
	41A	6.565	1.907	23	19	5.053	1.637	19	18	2.62**

$^{*}p < .05$
$^{**}p < .01$
$^{***}p < .001$

[1] The "t" values were computed for correlated means (matched pairs).

in the mean number of cards chosen and that this value is significant at the .05 level in problems 1, 19 and 25. In the experimental group, the difference is significant at the .05 level only for problem 1, while for problem 19, the difference is in the opposite direction. The variances for both college and high school freshmen, whether they belong to the control or experimental group, are always smaller at the post-testing session.

The over-all conclusions seem to point to the fact that in terms of these statistics there are not enough grounds to assume that the training sessions have had an effect in the experimental group when compared with the control group. Nevertheless, the control group received problems 1, 19, and 25 at the beginning of the experiment and had no further training (except for the Rosenweig and Rotter problems). It might be conceivable that while subjects in the control group remembered the problems, those in the experimental group, who had a large number of similar problems administered between the pre- and post-testing sessions, forgot them. This may be partially verified if the comparison of the control and experimental groups is performed for tests that are entirely new for both groups.

Problems 31D', 35B', are of type a, while 32F and 36F are of type b. If our assumptions are correct then the subjects of the control group should use more questions than those in the experimental group in the solution of these problems. This trend is clearly seen in Tables 12 and 13 for high school and college freshmen, respectively. The differences are significant in all cases at least at the .01 level (see t values in Tables 12 and 13), the only exceptions being problems 32F, where the difference is not significant though in the hypothesized direction, and 35B', college group, where the difference is at the .05 level.

Problem 32F (as presented in Appendix I) has the same presentation as that of problem 32, but a drastic change was introduced in the schema. As it were, the series in problems 32 A, B, C and D, move along the rows, while the series in 32F move along the columns. Experimental subjects had been trained in problems 32 and then confronted with the same figure but a schema that moved along a different dimension. This may explain the results obtained.

As far as the variances are concerned, only in problems of type a is there more homogeneity among the experimental than among the control subjects. This is only significant for problems 31D' and 35B' among college students (.001 for problem 31D' and .05 for problem 35B').

For problems 26 and 41A, that were entirely new, it is found (Tables 12 and 13) that the control group in all cases uses more cards than the experimental group. As indicated by the t test these differences are significant at the .01 level for both problems in the college freshmen, and at the .05 level for problem 41A in the high school freshmen.

The same general trend is observed for the variances, that is, control subjects are more heterogeneous than experimental subjects in all the tests, the only violator of this general trend being again problem 26, at the high school level.

Summarizing, it seems that:

a) High school freshmen change less in their observable behavior than college freshmen. The implications of this will be discussed later.

b) Training is more effective when problems of type a are used than when problems of type b are used, as shown by the improvement in problems 31D' and 35B'.

c) In entirely new problems (26 and 41A) experimental subjects perform "better" than control subjects (The case of problem 26 seems to be an exception to this trend only for the high school freshmen. But here several causes may be responsible, such as the fact that high school freshmen are less susceptible to changes and that the structure of problem 26 is less clear.).

d) For problems 1, 19 and 25, remembering may play a significant part in the results.

e) In general, performance in the experimental group is more homogeneous than in the control group.

2) Correct Answers

Performance in each problem and for each subject can be evaluated considering whether the problem has been solved correctly. This score is easy to obtain.

In Tables 5 and 6 the number of correct solutions for each problem administered during the training period are presented. The total values are presented in Tables 14 and 15.

The proportion of correct solutions does not change significantly for problems of type a when training with problems of type b has taken place and similarly, for problems of type b. Nevertheless, there are changes, already discussed, in number of questions used. This seems to enhance the likelihood of our suggestion that the correct solution of a problem and the process followed to reach this solution are not necessarily related.

Tables 8 and 9 give the number of correct solutions in the mid-test sessions. For the problems used during the mid-test session it was found, using Fisher's exact probability test that: problems 35A' and 32E do not show any significant change, at least at the .05 level, for either high school or college freshmen. Remembering that problem 35A' is a type a problem and 32E a type b problem, these results suggest that whether the training was in type a problems or type b problems, this training did not carry over in any significant manner to correctness of the solution in problems 35A' and 32E. However, for problem 36E, the number of correct solutions is greater in those subjects, either high school or college freshmen, that had been trained in type b problems. The value of Fisher's exact probabilities are respectively .036 and .00005. In problem 31C', it is found that for high school students the change is significant at the .048 level. That is, those subjects that were trained in type a performed better than those trained in type b.

Table 14

Total Values for the Number of Correct Solutions
in the Training Sessions
for High School Freshmen

Problems	Group a			Group b		
	Total Number of Problems	Correct Solutions	P. *	Total Number of Problems	Correct Solutions	P. *
Type a	155	107	.690	155	105	.677
Type b	153	106	.693	155	105	.677

* P. means proportion of correct solutions.

Table 15

Total Values for the Number of Correct Solutions
in the Training Sessions
for College Freshmen

Problems	Group a			Group b		
	Total Number of Problems	Correct Solutions	P. *	Total Number of Problems	Correct Solutions	P. *
Type a	168	139	.827	154	130	.844
Type b	134	114	.851	158	132	.835

* P. means proportion of correct solutions.

Table 16

Chi Square or Fisher's Exact Probability Test for Number of Correct Solutions in the Post-Testing Problems for Control and Experimental Groups in High School Freshmen

Problems	Control		Experimental		X^2	Fisher's Exact Probability
	Correct Solution	Wrong Solution	Correct Solution	Wrong Solution		
1	24	0	25	1		.520
19	18	6	22	4	.245	
25	22	2	23	3		.539
31D'	5	19	22	4	17.952***	
35B'	9	15	15	10	1.696	
32F	12	12	16	10	.287	
36F	2	22	20	6	21.126***	
26	9	15	14	12	.765	
41A	12	12	19	6	1.660	

*p $<$ .05
**p $<$ .01
***p $<$ .001

Table 17

Chi Square or Fisher's Exact Probability Test for Number of Correct Solutions in the Post-Testing Problems for Control and Experimental Groups in College Freshmen

Problems	Control		Experimental		X^2	Fisher's Exact Probability
	Correct Solution	Wrong Solution	Correct Solution	Wrong Solution		
1	21	2	18	1		.573
19	17	6	16	3		.336
25	19	4	16	3		.612
31D'	7	16	18	1	15.287***	
35B'	9	14	18	1	11.695***	
32F	10	13	13	6	1.703	
36F	3	20	18	1	24.604***	
26	9	14	11	8	.812	
41A	19	4	18	1		.237

*p $<$ .05
**p $<$ .01
***p $<$ .001

Simple inspection of Tables 10 and 11 indicates that the final test problems, 31D', 35B', 32F and 36F, do not show significant differences between groups a and b in problems of type a or of type b.

Comparing control and experimental groups in the final testing (Tables 16 and 17), it was found that there is no significant difference in the number of correct solutions given by the experimental and control subjects in problems 1, 19 and 25 or in problems 26 and 41A. Problem 32F behaves similarly.

Problems 31D' and 36F show that experimental subjects give more correct answers than controls for both high school and college freshmen. Problem 35B' shows the same trend but only for college students.

Summarizing, it seems that the study of the number of correct solutions does not show any striking results.

Many "byzantine" arguments could be brought in order to justify one or another type of results and all these arguments could be both right or wrong in terms of theoretical preconceptions. What emerges clearly is that, as far as these findings show, changes in the process (as indicated by number of questions used) are not related in any clear manner to the correctness of the solution. In relation to this last problem, several tests were performed, all of them pointing to the fact that no clear-cut evidence exists to assume that economy in questions asked is associated with correctness of the final solution, or vice versa.

3) Study of Group Performance Ellipsoids

The basic rationale underlying this method has been presented in several papers (Rimoldi, Devane, Haley, 1959; Rimoldi, Devane, 1961). In essence it involves the following steps:

a) Determine frequency with which each question has been asked by the group and find the ratio between this number and the total number of subjects in the group. This was defined as utility index. (Rimoldi, 1955).

b) Rank questions in terms of their utility indexes and cumulate these values for successive steps, thus obtaining a maximum curve (when cumulating from higher to lower utility indexes), and a minimum curve (when cumulating from lower to higher utility indexes).

c) Between maximum and minimum curves an ellipsoid is generated. The area of this ellipsoid will vary with the problems and the groups. If all the questions have the same utility indexes the ellipsoids degenerate into a straight line with slope proportional to the constant utility index. In the case in which some of the questions are asked by all subjects (utility index = 1.00) and the remaining by no subject (utility index = 0) the ellipsoid becomes a parallelogram. From this it can be inferred that the area of the ellipsoid is partially a function of the dispersion of the values of the utility indexes, i.e., agreement of the subjects on the usefulness of the questions asked.

d) The slope of the maximum curve indicates rate at which subjects increase their knowledge or decrease their ignorance. It can be shown that the amount of information that can be gained at any step is a constant fraction of the amount of information yet to be gained. Experimentally, it has been found for a large variety of problems that the function best fitting the maximum curve is,

$$Y_{max} = c(1 - e^{-bx}), \quad b > 0$$

and for the minimum curve,

$$Y_{min} = c(e^{bx} - 1), \quad b > 0$$

where y = summation of utility indexes at x steps in the sequence, c is the y asymptote and b the slope. The definition of b is:

$$b = \frac{y^1}{y - c},$$

where y^1 is the first derivative of the function. (Rimoldi, Devane, Haley, 1961).

e) The ratio between the area of the ellipsoid and the parallelogram may be taken as an indication of the amount of agreement among the subjects concerning the utility of each question, in relation to perfect agreement. (Rimoldi, Devane, Haley, 1961).

In this study a change has been introduced in step a) above, so that instead of using number of subjects in the group as denominator, the total number of selections by the group was employed. Thus the modified utility indexes are a linear function of the original ones, but the total sum of the modified utility indexes becomes in all cases 1.00. This facilitates the comparisons to be made, since the total number of selections made by the group is controlled. This will give a different view than the one used when comparing number of questions used. The rest of the procedure has not been altered. The values of "c" and "y" will be correspondingly changed by the operation used to change from number of subjects to total number of selections.

For every problem in the pre-test session, mid-test session and post-test session, the ellipsoids and corresponding parallelograms were computed. The corresponding figures are given in Appendix II. Tables 18 and 19 give the values of the ratios between the ellipsoids and the parallelograms as well as the significance of differences between the maximum curves for different problems computed using the Kolmogorov-Smirnov Test (Goodman, 1954).

For the pre-testing session (problems 1, 19 and 25), there are no significant differences between the control and experimental groups in either the high school freshmen or the college freshmen.

At the mid-testing session the ellipsoids for problems 31C' and 35A' are significantly greater in subjects of group a than in subjects of group b. On the other hand, for problems 32E and 36E no significant difference is found. Remembering that the odd numbered problems are of type a, and that group a was trained first in problems of type a, it is clear as indicated by the values shown in Tables 18 and 19, that training in problems of type a is more effective than training in problems of type b. This training carries over to problems of the same type.

At the end of the experiment when all the subjects have been equally trained in problems of type a and b, the differences should disappear, since both groups a and b have then had equal training. This is what happens as it can be observed in Tables 18 and 19. However there is one exception and this refers to problem 35B', in college freshmen, which shows a higher ratio in group a than in group b (ellipsoids significantly different at .05 level).

Comparing the ellipsoids for the control and the experimental groups, no differences were found in the performance of tests 1, 19 and 25, either at the college or at the high school level. This agrees with the previous findings reported in this study.

In problems 31D' and 35B' the experimental groups show a higher ratio than the corresponding control groups. The values of the ellipsoids are significantly different beyond the .01 level (Tables 18 and 19). The ellipsoids for problem 32F are not statistically significant. In problem 36F the difference is significant at the .01 level in favor of the experimental group among the college freshmen.

Problem 26 shows no statistically significant differences, and problem 41A is significantly higher (.05 level) for the experimental group among the college students.

In summary, if these ratios are interpreted as indications of agreement among the group the previous findings indicate that:

a) Problems of type a are more effective than those of type b.

b) The college group "improves" more under training than the high school group.

c) Training has produced a change in performance in all the tests with the exception of problems 32F among college freshmen and 32F and 36F in high school subjects.

Table 18

Ratios Between the Ellipsoids and the Parallelograms and Values of the Kolmogorov-Smirnov Test of Significance for the Pre-Testing, Mid-Testing, and Final-Testing in High School Freshmen

Problems		Ratio		Ratio		χ^2 Kolmogorov-Smirnov	
		Control	Experimental	Group a	Group b	Control/Exper.	Group a/Group b
Pre-Testing	1	.2000	.4535			1.71	
	19	.6824	.6324			1.28	
	25	.4446	.4534			.27	
Mid-Testing	31C'			.8141	.5202		7.94**
	35A'			.8108	.7454		4.82*
	32E			.5982	.5938		.34
	36E			.5026	.2836		.11
Final Testing	1	.5132	.6926			.81	
	19	.7788	.8269			.20	
	25	.6105	.5546			1.04	
	31D'	.5833	.8904	.8856	.9144	10.92**	.57
	35B'	.6306	.7762	.7883	.8108	7.80**	.32
	32F	.4643	.3596	.2806	.5273	.01	1.24
	36F	.4203	.2945	.3485	.3036	.52	.04
	26	.6084	.4694			1.31	
	41A	.6239	.6093			.73	

*$p < .05$
**$p < .01$

Table 19

Ratios Between the Ellipsoids and the Parallelograms and Values of the Kolmogorov-Smirnov Test of Significance for the Pre-Testing, Mid-Testing, and Final-Testing in College Freshmen

Problems		Ratio		Ratio		X^2	Kolmogorov-Smirnov
		Control	Experi-mental	Group a	Group b	Control/Exper.	Group a/Group b
Pre-Testing	1	.5661	.4643			.06	
	19	.8860	.6921			.64	
	25	.6252	.5767			3.06	
Mid-Testing	31C'			.8432	.5820		6.27*
	35A'			.8072	.6343		4.63*
	32E			.6192	.4863		.98
	36E			.5408	.5456		3.25
Final Testing	1	.4476	.6384			2.14	
	19	.8341	.8390			.31	
	25	.6919	.7115			.70	
	31D'	.6990	.9021	.9151	.9428	12.55**	.26
	35B'	.6381	.8419	.9519	.7737	10.98**	6.06*
	32F	.5070	.4649	.5536	.5208	.01	.05
	36F	.4225	.4268	.5500	.3842	10.09**	1.34
	26	.7180	.7874			3.08	
	41A	.6733	.7978			5.84*	

*$p < .05$
**$p < .01$

d) For the entirely new problems, while problem 26 shows no effects of training, problem 41A shows a change at the college level.

Again it becomes more clear that problems 32F and 26 may not be good problems for our purposes, and that college freshmen, on the whole, are more susceptible to change than high school freshmen.

d) Analysis of Individual Curves at the Group and Individual Levels

The scoring methods thus far described do not take into consideration the order in which questions are asked. For a given test and a given group it is possible to count the number of times that each question has been asked by the group in each order. The number of times a question has not been asked can also be easily determined. This we have called 0 order. Each one of these frequencies can be expressed as a proportion of the total number of selections and non-selections made by the group. This denominator is obtained by multiplying the number of questions in the test by the number of subjects. The proportions in each table by definition will then necessarily add to 1.00.

The performance of every subject can be graphically represented by adding these proportions (observed) according to the value of the questions in the observed order. The total sum of these values is a summary statement of the tactics used by the subject (Rimoldi, 1961; Rimoldi, Devane, 1961; Rimoldi, Haley, 1962a).

If the procedure just outlined is applied, then every subject is scored in terms of his own group. Nevertheless, if we desire to obtain performance curves on the basis of other criteria, then the operation described will be the same but the values corresponding to each question in each order will probably be different, since they have been obtained from the performance of another -criterion- group or on the basis of a predetermined condition.

The proportions just mentioned can be presented in the form of a matrix having the columns represent question and the rows represent order. Each matrix has then a set of values that will vary in terms of the group used for their computation. Also matrices assuming randomness, or any other hypothesis, can be built. In an unpublished study by Rimoldi and Georgas, probability values for the occurrence of a specific question in a specific order have been calculated.

The same performance may be scored in terms of the values of each one of the matrices, or in terms of the differences between corresponding cells of any two matrices. Thus, if we are interested in comparing "observed" performance in terms of the group to which the subject belongs, to performance expected in terms of another group, or in terms of randomness, etc., the differences between the corresponding cells in the two matrices under consideration will be smaller the more similar the values. If they are identical they will be 0 and the graphical representation of this observed minus expected (O - E) performance will give, by accumulating the successive questions asked, a horizontal line at the 0 ordinate. Whenever there is a difference, a departure from the 0 line will appear. Finally, the total value obtained by accumulating all the corresponding (O - E) values will give a summary statement related to the over-all departure of a tactic scored in terms of the difference between observed and expected values.

This means then that for every subject:

a) Observed performance curves can be obtained.

b) Expected performance curves can be obtained.

c) The difference (O - E) curve can be obtained.

d) A summary statement for the observed tactics can be obtained.

e) A summary statement about (O - E) tactics can also be obtained.

This can be done also for questions not asked, that is in 0 order. But since here we have no way of estimating in which order the questions not asked would have been asked if this were possible, then no curves can be drawn, though the accumulation of the values of the questions not asked is possible. This holds both for the observed and expected and (O - E) values. That is, every performance can be expressed as a) what the subject does, and b) what the subject does not do.

The previous type of evaluations are essentially based on averages. But approaches of this kind eliminate, by their very definition, individual differences, or at least make it difficult to observe them without the bias introduced by the use of averages. One of our aims has been to define norms that would not depend on average group performance, but rather on the properties of the problems administered to the subjects.

In trying to understand the specifications needed to use a scoring approach not based on averages, the concepts of "schemata" and "content" were derived (Rimoldi, Haley, Fogliatto, Erdmann, 1962, unpublished). Schemata are represented by the formal properties or set of relationships embodied in the problem, and content are the objects used to realize the problems, i.e., objects, numbers, etc.

Most of the problems used in this research were prepared by inventing first the appropriate schemata and then introducing, for the same schema, different contents. The schemata were then varied in difficulty by the experimenter. This usually represented the introduction of a larger number of relationships of the same type or new type of relationships not yet dealt with in previous schemata. (In Appendix I and Figures 1 to 6 inclusive, the schemata are given).

Several scoring methods may be developed. Two of them will be presented.

The study of the schemata makes it possible to establish the scoring norms for each problem in terms of the sequence of logical relationships involved. This is a relatively easy matter once the structure of the problem is known. For instance, for the schema presented in Figure 1, problem 31A, the rationale for the scoring norms is presented in Figure 7.

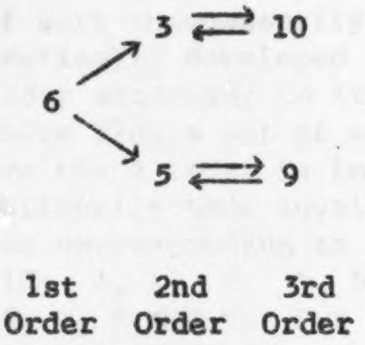


Figure 7

Question 6 should be first in all the tactics. It may be followed by 3 and 10 or by 5 and 9. Both 3 and 10 can be interchanged so that each one of them may be asked in 2nd or 3rd order, and similarly 5 and 9. The four possible tactics are presented in Table 20.

Table 20 *

Four Possible Tactics for Problem 31A

	Questions									
Tactics	1	2	3	4	5	6	7	8	9	10
a			2			1				3
b			3			1				2
c					2	1			3	
d					3	1			2	

* (The entries represent order of choice.)

When scoring the performance of a given subject, a value of 1.00 or 0 may be assigned to each successive choice according to its agreement with the norms. For instance, if we use tactic a̲ and a subject selects questions 6, 3, 10 in this order then he will obtain a score of 1.00 for each question since he agrees perfectly with the theoretically developed tactic.

If the subject selected questions 6, 10, 3, and we score in terms of tactic a̲, (Table 20), then he will only obtain a score of 1.00 for question 6, but 0 for questions 10 and 3, though, if he were scored in terms of tactic b̲, his score would be 3. In order to be fair, every subject should then be scored in terms of all the theoretically developed tactics. This will increase the amount of work unnecessarily. It was thus decided to pool together all the theoretically developed tactics and assign a weight to each question in each order according to its frequency of occurrence in that order. Thus, for problem 31A, a set of weights could be that given in Table 1, Appendix III, where the 0 order is included and the sum of all the entries in the table is arbitrarily made equal to 1.00, in order to have the same total in all the tables corresponding to all the problems. The possible tactics are: 6, 3, 10; 6, 10, 3; 6, 5, 9; and 6, 9, 5. Questions not in the tactics are 1, 2, 4, 7 and 8. For instance, question 6 appears in all the tactics in first position which makes a total of 4, that averaged in terms of tactics gives a value of 1.00. Since there are 10 cards, the value finally given to 6 will be .10. Question 10 may appear in order 2 or 3, in all the possible tactics as defined. Thus, adding the corresponding value and dividing by the total number of tactics multiplied by the total number of cards in the problem, we obtain a value of .05 as shown in the Σ row of Table 1, Appendix III. But, since it can be located in orders 2 or 3 the corresponding value for this card in these orders will be respectively .025 and .025. Since having 10 cards and, under the assumption of equal probability of each card being chosen, the Σ column should be, for all the cards .10. Then, the Σ_0, that is, the value for the zero position, will be .05.

In order to score the performance of a subject and obtain his performance curve, several approaches may be followed. Some of these will be discussed.

A) Add the values given in the corresponding table for successive choices. Thus, for the tactic 9, 6, 10, 5, we obtain, from Table 1, Appendix III: 0 + . 0 + .025 + 0 = .025. The performance curve will then show a plateau at 0 for the first two questions rising to .025 in the third question and adding nothing for the fourth question.

Tactic 6, 3, 10, in terms of Table 1, Appendix III, will be: .100 + .025 + .025 = .150. This method is conservative in the sense that if a subject has chosen the question according to the prescribed tactics in the order defined, but other irrelevant questions have also been asked, then this may reduce his score to 0. For instance, a subject choosing 1, 6, 4, 3, 10, since 6 does not occur in 2nd order and 3 does not occur in 4th order and 10 does not occur in 5th order, will obtain a total score of 0.

Nevertheless, it may be said that the subject did not realize the problem right away and that after asking question 1, reorganized his thoughts and realized that question 6 should have been asked first. The same reasoning can be used for the other irrelevant questions.

B) From the observed sequence, pull out the irrelevant questions keeping the relevant ones in their original position, but renaming them following the sequence of natural numbers, 1, 2, 3,.

For instance, the sequence 1, 6, 3, 8, 2, 10, (see Figure 7) becomes 6, 3, 10, where 6 is first, 3 second, and 10 third, that is: .100 + .025 + .025 = .150.

If, after pulling out the irrelevant questions, the sequence that remains only represents partially any of the predefined tactics, proceed by considering only those items that follow the sequence of the predefined tactic.

For instance, the sequence 1, 3, 6, 2, 8, 10, becomes 3, 6, 10. A person asking for question 3, in terms of the norms indicates worse planning of his thinking performance and this should be considered in his scoring. Nevertheless, the fact that his next question was 6 followed by 10, indicates that some reshaping has taken place. There is, of course, the possibility that this is purely accidental but it would be almost impossible to know it. The subject's score will then depend only on questions 6 and 10, that is: .100 + .025 = .125.

The sequence 1, 3, 10, 2, 8, 6, becomes 3, 10, 6. For the same reason stated above, only 6 could be given credit, that is, .10.

Now, if the sequence involves several of the predefined tactics, in part or in whole, then after eliminating the irrelevant questions, give credit to those questions that will give the maximum score. This, of course, gives the benefit of the doubt to the subjects. For instance the sequence 1, 6, 5, 10, 2, 3, after eliminating the irrelevant questions, becomes 6, 5, 10, 3, from where sequences 6, 5 and 6, 10, 3 can be obtained. Give credit for sequence 6, 10, 3, if this will give a higher score than 6, 5. For instance, sequence 6, 10, 3 gives .100 + .025 + .025 = .150, while sequence 6, 5 gives .100 + .025 = .125.

After adding the weights for the sequence, divide the total amount by the total number of questions asked. This implies that any irrelevant question is similar to any other irrelevant question. This implies also that not only questions defined as irrelevant in terms of the predefined tactics, but also questions that are relevant but asked in the wrong order, are given a score of 0. This system of scoring seems to fulfill the notion that both question and order are important as well as planning the attack to the problem.

Notice that using the "pulling out" method, the sequence 9, 6, 10, 5, given in A) above becomes 6, 10 or 6, 5, that is, in both cases .100 + .025 = .125.

Dividing the value of the sequences given above by the total number of questions used, we obtain:

Sequence 1, 6, 3, 8, 2, 10 = $\frac{.150}{6}$ = .025

Sequence 1, 3, 6, 2, 8, 10 = $\frac{.125}{6}$ = .021

Sequence 1, 3, 10, 2, 8, 6 = $\frac{.100}{6}$ = .017

Sequence 1, 6, 5, 10, 2, 3 = $\frac{.125}{6}$ = .021

*Sequence 9, 6, 10, 5 = $\frac{.125}{4}$ = .031

One of the maximum tactics will be 6, 5, 10, that is $\frac{.150}{3}$ = .50, but 6, 2, 5, 10, will be $\frac{.150}{4}$ = .375. That is, question 2 has reduced the final value from .50 to .375.

The performance curves that represent the subject's approach to the problem are built by accumulating the values of all the questions asked in the corresponding order, as indicated in A). (Tables in Appendix III).

Thus, a given tactic will have a unique performance curve but the numerical representation will depend on whether methods A), or B) above are used.

Though method B) has been explored in some cases, the bulk of our data was studied using method A).

For each subject in each test his performance curve was drawn and the score was computed as indicated in the previous sentence. *

Performance curves were drawn for each experimental subject in problem 31A, 31B, 31C, 31D, 33A, 33B, 33C, 33D, 35A, 35B, 35C, 35D, 32A, 32B, 32C, 32D, 34A, 34B, 34C, 34D, 36A, 36B, 36C, 36D and in the college experimental, 37A, 37B, 38A, and 38B.

The tables as obtained from the schemata for each one of these problems are given in Appendix III. Also, for each subject, whether control or experimental, performance curves were built for problems 26 and 41A. These performance curves were obtained by a) observed group performance (0 curves) and b) score following the schemata. These two curves for each single subject in each problem were plotted together on the same graph.

It would be impossible to describe every one of the curves plotted using both methods, and thus we will limit our analysis to a few of them. These

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* Assume that in a problem, in terms of the schemata, the following sequences are found, 6, 3, 9, and 5, 4, 2, where in the first tactic 3 and 9 may be interchanged. The table of tactics will then look as follows.

Questions

Tactics	1	2	3	4	5	6	7	8	9	10
a			3			1			2	
b			2			1			3	
c		3		2	1					

The entries in this table represent order of choice

Considering that both, 6, 3, 9, and 5, 4, 2, are equally good, then using the method described, a subject using tactic 5, 4, 2 will be punished in his final score, which will be lower than the one corresponding to 6, 3, 9. In order to avoid this, a system of weighting was derived, such that, for instance, question 6 and question 5, in first order would have the same value. Similarly, for the other questions. Most of our subjects were rescored using this approach, but the differences, when compared with the scoring method described in the body of the text, were minimal, and thus unless it is explicitly stated so, the results presented in this study are not obtained by using the weighted approach. For some problems it might conceivably be advantageous to use the weighted approach.

curves are given in Figure 8, and their study will help to understand more clearly the properties of both scoring systems.

In Figure 8, the performance of a subject in problem 31A (See Appendix I) is given. The tactic was 9, 6, 10, 5. The solid line performance curve is the one obtained using the observed (group) values and the broken line curve is obtained using schema values. The subject solved the problem. In terms of the schema he is on the zero line for the first two selections. The broken line curve shows two plateaux, one corresponding to the first few questions asked and the other one corresponding to the last question asked. Nevertheless, the observed performance curve shows a continuous increase, though differently accelerated in different parts of the curve. Judging by this curve, we would be tempted to conclude that at all steps during the process the subject was accumulating "knowledge", but when his performance is analyzed, considering the sequence as obtained from the schema of the problem, it is found that his planning was rather poor. That is, in order to solve the problem, as he did, he probably had to throw the information he obtained from the questions he asked, back and forth to complete it.

Next, let us take the curves that correspond to the same subject in problem 31B, which has the same schema as problem 31A, but an abstract content (see Appendix I). Here, the sequence used was 8, 9, 5, 3, 1, 2, 4, 7. According to the schema, the sequence should be 1, 7, 2, 9 or 1, 7, 4, 8. The subject did not solve the problem. If attention is paid to the observed curve (group norms) one would be tempted to conclude that his performance showed a progressive improvement. Nevertheless, the broken line shows a completely different picture; it now raises from the zero line. The subject seems to have never done any clear-cut planning of his procedure. As a matter of fact, he asked questions of both sequences, 1, 7, 2, 9 and 1, 7, 4, 8, without any clear idea of what and when to ask.

This is still more dramatic for the same subject in problem 31D. This problem (see Appendix I) reaches the highest degree of abstraction within the same schema. The subject did not solve it. His sequence was 2, 4, 6, 7. In terms of the schema he should have used sequence 6, 2, 4, 7 or 6, 2, 3, 8. The observed performance is very impressive, while the performance in terms of the schema is very poor. Again, though the subject asked all the questions required, he showed little planning in his sequence. His plateau is on the zero line for the first three selections.

The remaining curves show the performance of another subject in Problem 33C *, based on the performance of a subject who used the sequence 2, 9, 10, 6. The sequences derived from the schema can be: 2, 9, 10, 6 or 2, 9, 12, 11, 6. Notice that, in terms of the observed group values, the

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* The tables for observed values are not presented in this report. They are available upon request to the authors.

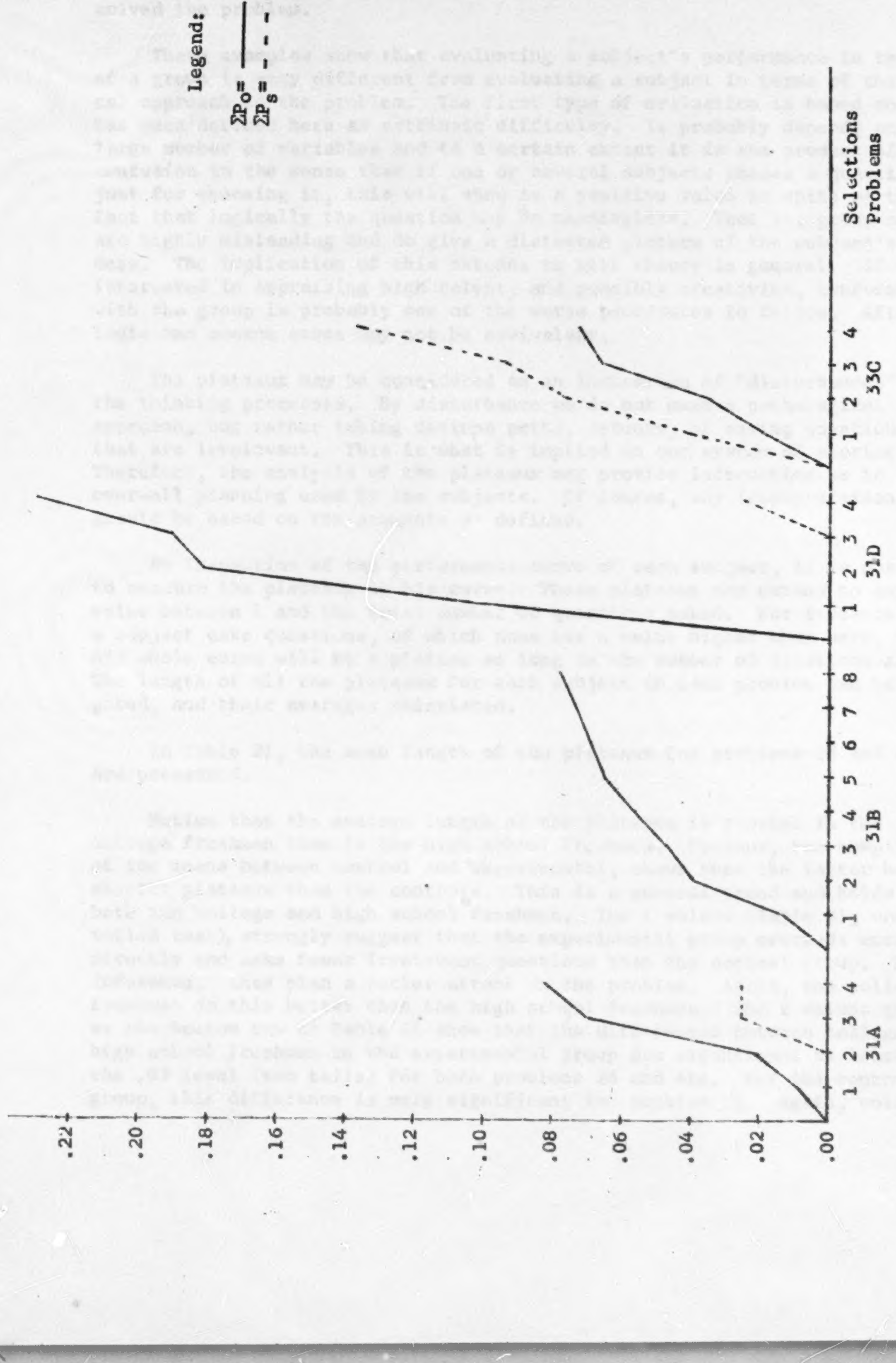

Figure 8. Observed (Group Norms) and Weighted (Schemata Norms) Individual Performance Curves of a Special Subject for Specified Problems.

curve is "worse" than the one obtained using the schema values. The subject solved the problem.

These examples show that evaluating a subject's performance in terms of a group is very different from evaluating a subject in terms of the logical approach to the problem. The first type of evaluation is based on what has been defined here as extrinsic difficulty. It probably depends on a large number of variables and to a certain extent it is the product of confusion in the sense that if one or several subjects choose a question, just for choosing it, this will show as a positive value in spite of the fact that logically the question may be meaningless. That is, group norms are highly misleading and do give a distorted picture of the subject's process. The implication of this extends to test theory in general. If we are interested in appraising high talent, and possibly creativity, conformity with the group is probably one of the worse procedures to follow. After all, logic and common sense may not be equivalent.

The plateaux may be considered as an indication of "disturbances" in the thinking processes. By disturbance we do not mean a pathological approach, but rather taking devious paths, detours, or asking questions that are irrelevant. This is what is implied in our system of scoring. Therefore, the analysis of the plateaux may provide information as to the over-all planning used by the subjects. Of course, any interpretation should be based on the schemata as defined.

By inspection of the performance curve of each subject, it is possible to measure the plateaux in his curve. These plateaux may extend to any value between 1 and the total number of questions asked. For instance, if a subject asks questions, of which none has a value higher than zero, then his whole curve will be a plateau as long as the number of questions asked. The length of all the plateaux for each subject in each problem can be computed, and their averages calculated.

In Table 21, the mean length of the plateaux for problems 26 and 41A, are presented.

Notice that the average length of the plateaux is shorter in the college freshmen than in the high school freshmen. Further, the comparison of the means between control and experimental, shows that the latter have shorter plateaux than the controls. This is a general trend and holds for both the college and high school freshmen. The t values (Table 21, one tailed test), strongly suggest that the experimental group proceeds more directly and asks fewer irrelevant questions than the control group. By inference, they plan a better attack on the problem. Again, the college freshmen do this better than the high school freshmen. The t values given at the bottom row of Table 21 show that the differences between college and high school freshmen in the experimental group are significant at least at the .05 level (two tails) for both problems 26 and 41A. For the control group, this difference is only significant for problem 26. Again, college students seemed to change more than high school freshmen, and problem 26 differentiates better than problem 41A.

Table 21

Mean Length of Plateaux for Problems 26 and 41A in the
Final Testing for High School and College Freshmen

	Problem 26			Problem 41A		
	Control	Exper.	"t" value (one tail)	Control	Exper.	"t" value (one tail)
High School	6.136	5.00	1.70[1]	4.409	3.727	1.34[1]
College	4.188	2.938	2.10*	3.312	2.312	1.39[1]
"t" value (two tails)	2.09 *	2.17 *		1.30	2.33 *	

* $p < .05$
[1] $p < .10$

The length of the plateaux was measured for Tests 31 (A, B, C, D), 33 (A, B, C, D) and 35 (A, B, C, D) in each subject during the training sessions.

An analysis of variance was performed separately for the high school and for the college freshmen.*

The results were similar in both cases, showing that the schemata corresponding to tests 31, 33, and 35, and the content corresponding to forms A, B, C, D, contributed significantly (over the .01 level) to the total variance. A significant F ratio (.01 level) was also obtained for the interaction schemata-content, in the college freshmen, and for the interaction individuals-schemata, in the high school freshmen.

That is, schemata and content are both significant sources of variation This helps to verify some of the assumptions used in building the tests, such that schemata could be made progressively more complex and content could be made progressively more unfamiliar.

The mean length of the plateaux for high school and college freshmen is presented in Tables 22 and 23. A graphic representation of these tables is given in Figures 9 and 10.

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* These analyses of variance are available upon request to the authors.

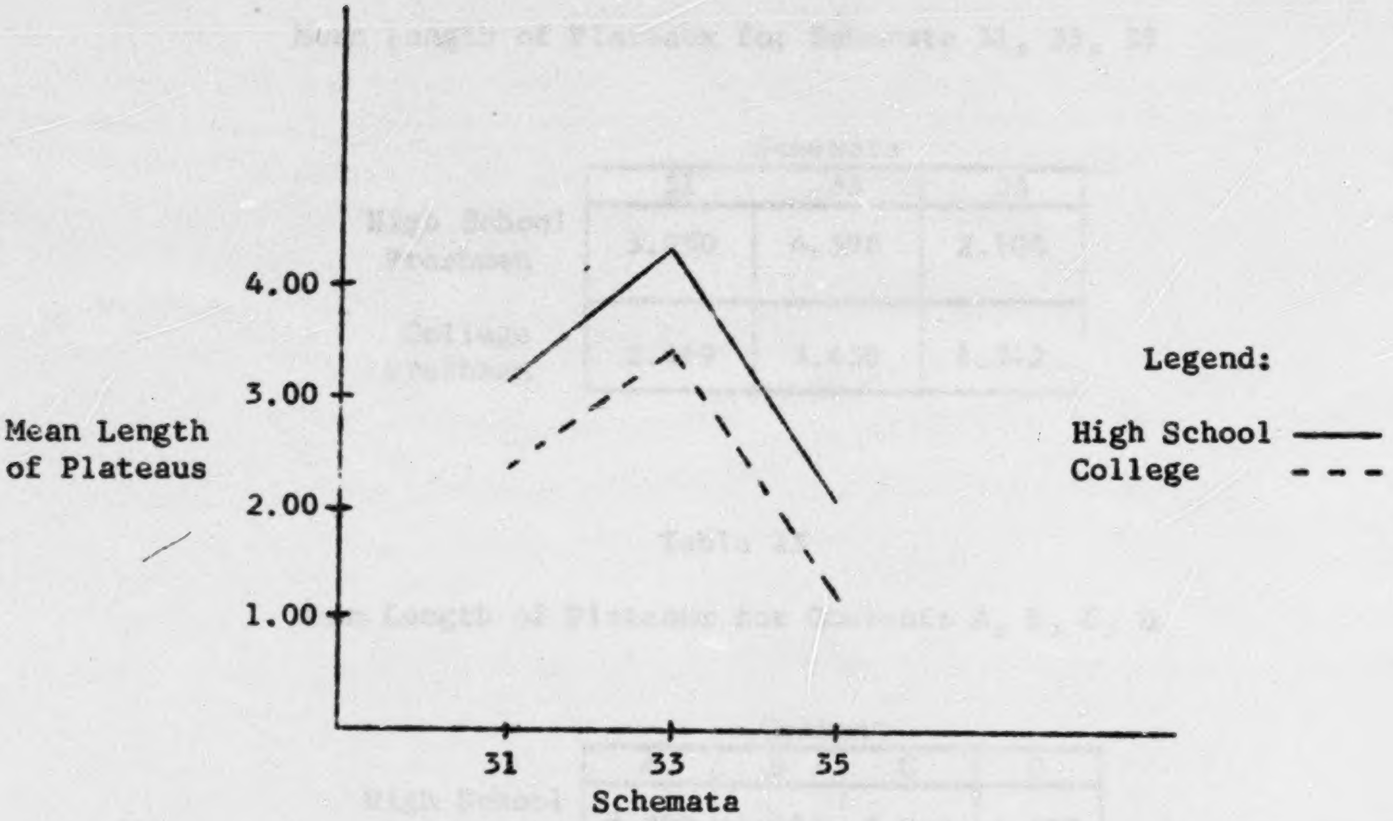


Figure 9. Mean Length of Plateaus of High School and College Subjects on Problems 31,33, and 35.

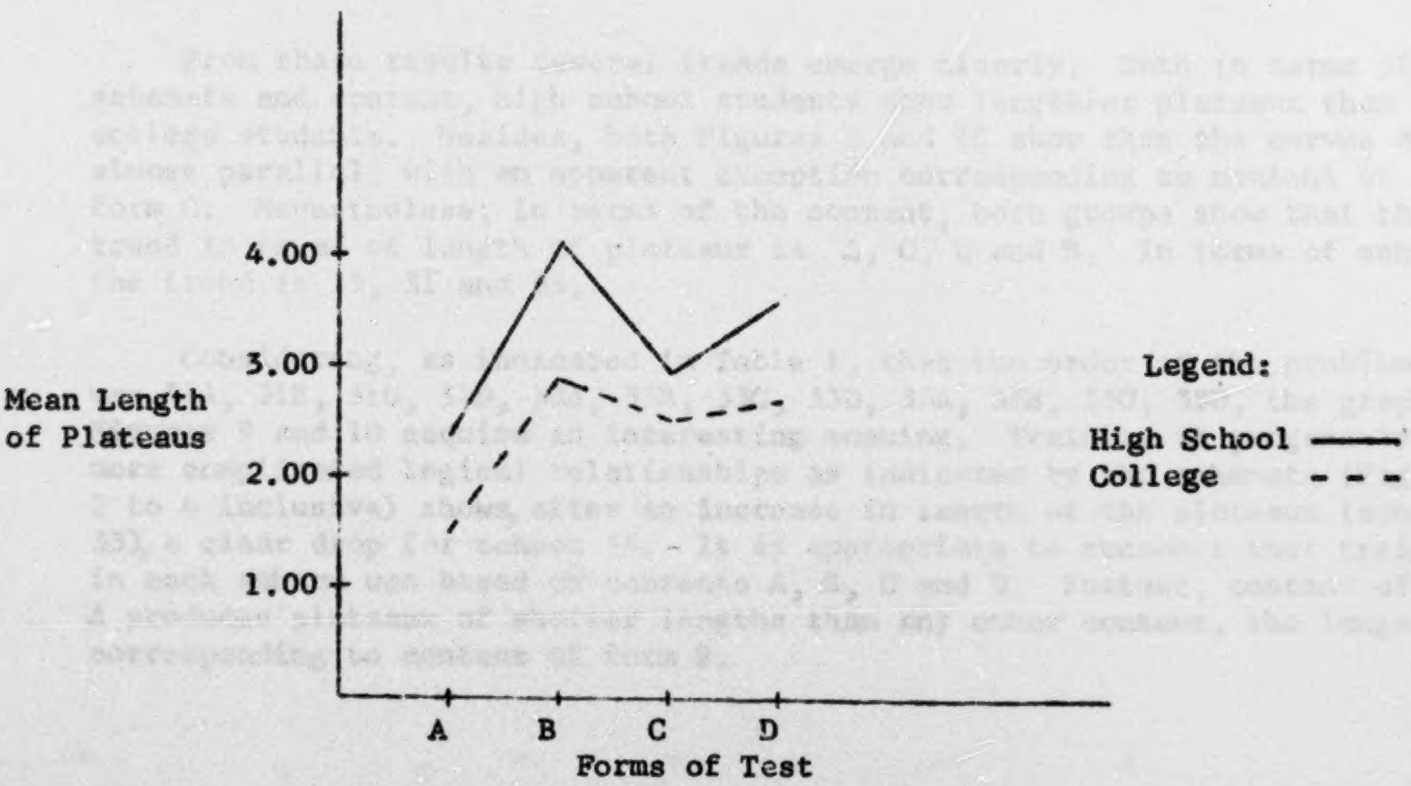


Figure 10. Mean Length of Plateaus of High School and College Subjects on the Four Forms of the Tests.

Table 22

Mean Length of Plateaux for Schemata 31, 33, 35

	Schemata		
	31	33	35
High School Freshmen	3.250	4.396	2.104
College Freshmen	2.469	3.438	1.312

Table 23

Mean Length of Plateaux for Contents A, B, C, D

	Content			
	A	B	C	D
High School Freshmen	2.389	4.083	3.000	3.528
College Freshmen	1.500	2.958	2.542	2.625

From these results several trends emerge clearly. Both in terms of schemata and content, high school students show lengthier plateaux than college students. Besides, both Figures 9 and 10 show that the curves are almost parallel, with an apparent exception corresponding to content of form C. Nevertheless, in terms of the content, both groups show that the trend in terms of length of plateaux is A, C, D and B. In terms of schemata, the trend is 35, 31 and 33.

Considering, as indicated in Table 1, that the order of the problems was 31A, 31B, 31C, 31D, 33A, 33B, 33C, 33D, 35A, 35B, 35C, 35D, the graphs of Figures 9 and 10 acquire an interesting meaning. Training in progressively more complicated logical relationships as indicated by the schemata (Figures 2 to 4 inclusive) shows, after an increase in length of the plateaux (schema 33), a clear drop for schema 35. It is appropriate to remember that training in each schema was based on contents A, B, C and D. Further, content of form A produces plateaux of shorter lengths than any other content, the longest corresponding to content of form B.

Because of the order of administration of the problems, these findings ought to be appropriately weighted since, before beginning problems with schema 33, the subjects have been trained with four problems with schema 31, and before starting with problems with schema 35, the total number of previous training problems had been eight. There is not the same monotonic relationship between content and number of previous training problems. For instance, content B was present in the 2nd, 6th and 10th training problem. This makes the interpretation of Figure 10 less clear than the interpretation of Figure 9.

In Tables 24 and 25, the mean length of the plateaux for each problem is presented.

Table 24

Mean Length of Plateaux (High School Freshmen)

Schemata	Content			
	A	B	C	D
31	1.25	4.67	3.08	4.00
33	4.00	4.92	4.33	4.33
35	1.92	2.67	1.58	2.25

Table 25

Mean Length of Plateaux (College Freshmen)

Schemata	Content			
	A	B	C	D
31	1.00	3.88	2.12	2.88
33	2.00	4.00	3.75	4.00
35	1.50	1.00	1.75	1.00

Figures 11 and 12 represent the values given in Tables 24 and 25, respectively.

Comparing Figures 11 and 12, it emerges clearly that the curves for the same schema followed the same trend in high school and college freshmen. Schema 31, which was present in the first four problems administered, shows the intervariation for different contents. Schema 35 is the one showing less variation and so is schema 33 in high school students. In college students, there is a drastic change only between forms A and B.

Inspection of these graphs points to the fact that, while there is for schema 31 and 33 a clear interaction with content, in schema 35 the familiarity of the content is less important in determining the length of the plateaux. As is were, training produces a homogeneity in performance, reducing the effects of content familiarity.

In order to clarify the interesting points that this suggests, further work is being performed. It will not be reported here because of insufficiant time to complete it.

Nevertheless, if a certain amount of cautious interpretation is allowed, changing content from very concrete (A) to abstract (B) produces a noticeable change in performance. In general, familiarity with the content seems to be inversely related to length of plateaux, up to a critical training period. Plateaux, as explained before, may indicate "defective" planning. Again, the results seem to suggest that increasing the complexity of the schemata (31, 33, 35) tends to lengthen the plateaux up to a point in which training, as it were, becomes effective and overshadows the effects of this complexity. Of course we do not know what type of exact relationship holds between these variables. The possibility of verifying these points would require a design slightly different from the one used in this study. A further point to be mentioned is that the type of analysis that preceeds was not used for problems 32 to 36 inclusive or for group b who took problems of type b first.

Large numbers of individual performance curves have been drawn for each sugject in all the problems administered during the training session using the scores based on group norms and schemata. The study of the plateaux as training proceeds as well as other characteristics of the curves is now in progress. Changes in the curves have been observed as training proceeds. The meaning of these changes is not yet totally clear. No further evaluation of the performance curves will be made, other than the study already reported.

e) Convex Sets

The rationale behind this method for the study of processes has already been published (Rimoldi, 1961; Rimoldi, Devane, 1961; Rimoldi, Haley , 1962a)

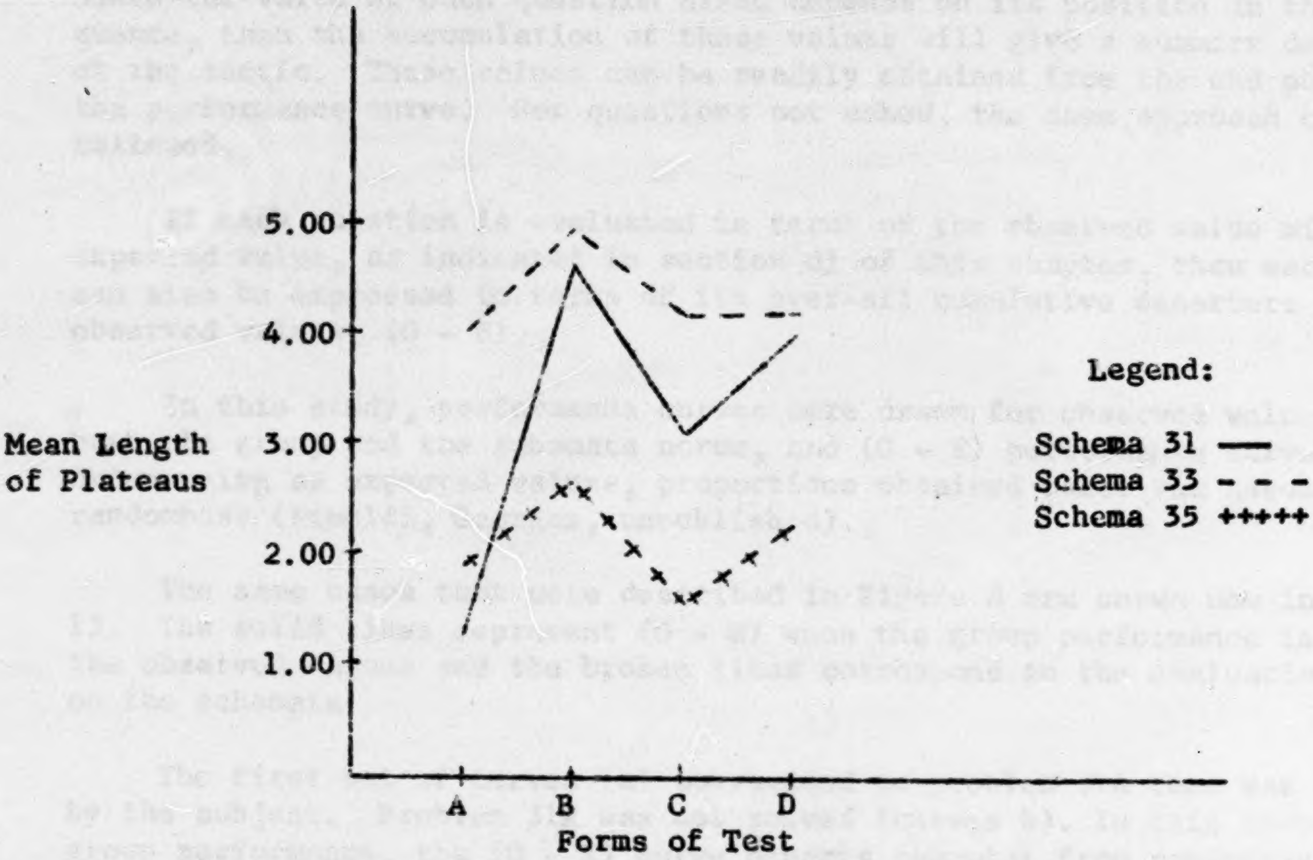


Figure 11. Mean Length of Plateaus of High School Subjects on Problems 31,33,and 35 for the Four Forms of Each Problem.

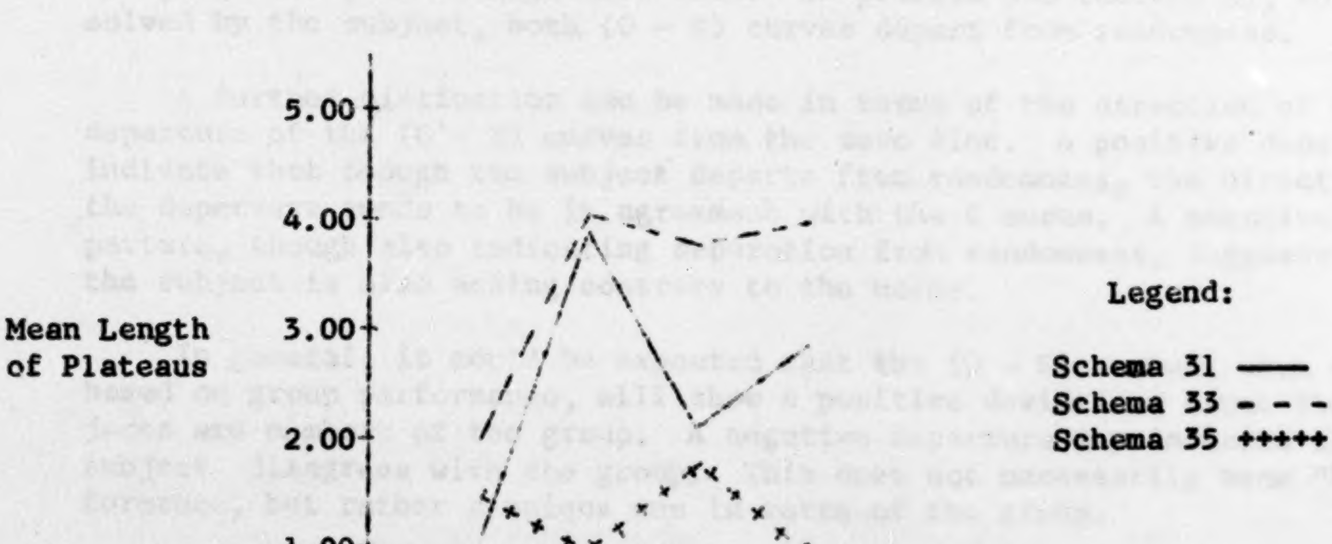


Figure 12. Mean Length of Plateaus of College Subjects on Problems 31,33, and 35 for the Four Forms of Each Problem.

Tactics have been defined in terms of the sequence followed in solving a problem. Thus, for every problem, there is a limited number of tactics. Since the value of each question asked depends on its position in the sequence, then the accumulation of these values will give a summery description of the tactic. These values can be readily obtained from the end point of the performance curve. For questions not asked, the same approach can be followed.

If each question is evaluated in terms of the observed value minus the expected value, as indicated in section d) of this chapter, then each tactic can also be expressed in terms of its over-all cumulative departure from the observed values, (O - E).

In this study, performance curves were drawn for observed values, using both the group and the schemata norms, and (O - E) performance curves were drawn using as expected values, proportions obtained under the assumption of randomness (Rimoldi, Georgas, unpublished).

The same cases that were described in Figure 8 are shown now in Figure 13. The solid lines represent (O - E) when the group performance is used for the observed values and the broken lines correspond to the evaluation based on the schemata.

The first set of curves (a) correspond to problem 31A that was solved by the subject. Problem 31B was not solved (curves b). In this case, using group performance, the (O - E) curve departs somewhat from randomness while by using the schemata, the (O - E) curve shows a progressive decrement, thus departing from randomness more than the group performance curve. For problem 31D (curves c), which also was not solved by the subject, the (O - E) curve departs very markedly from randomness when using group norms, while when using schemata, it remains near zero. In problem 33C (curves d), which was solved by the subject, both (O - E) curves depart from randomness.

A further distinction can be made in terms of the direction of the departure of the (O - E) curves from the zero line. A positive departure may indicate that though the subject departs from randomness, the direction of the departure tends to be in agreement with the O norms. A negative departure, though also indicating separation from randomness, suggests that the subject is also acting contrary to the norms.

In general, it could be expected that the (O - E) curves, when O is based on group performance, will show a positive deviation, since the subjects are members of the group. A negative departure may indicate that the subject disagrees with the group. This does not necessarily mean "bad" performance, but rather a unique one in terms of the group.

In all the curves for all the subjects used in this study, the tendency of the (O - E) curves is in the positive direction, when using group norms. Returning to Figure 13, notice that for curves a, b and c, the trend des-

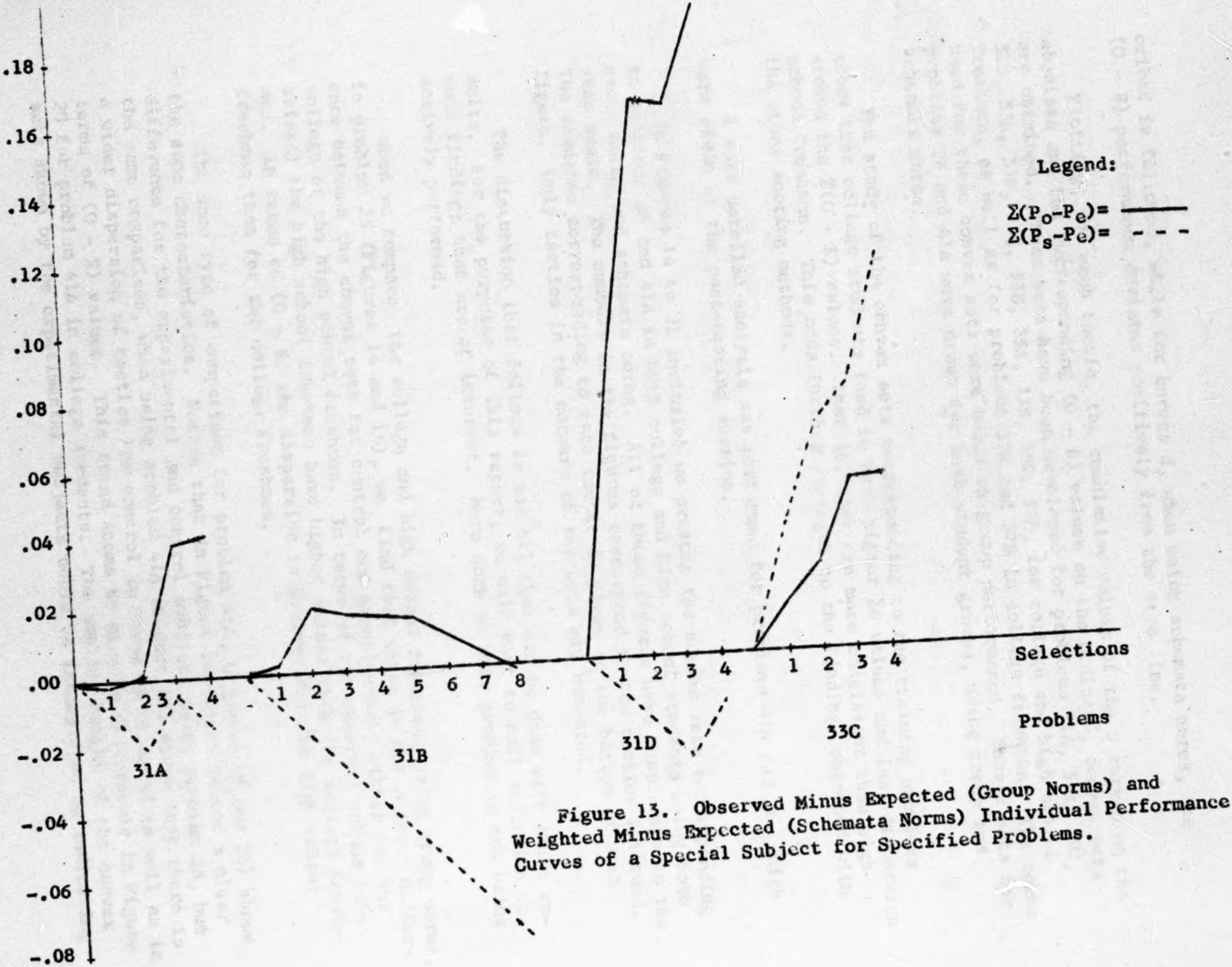


Figure 13. Observed Minus Expected (Group Norms) and Weighted Minus Expected (Schemata Norms) Individual Performance Curves of a Special Subject for Specified Problems.

cribed is followed, while for curves d, when using schemata norms, the (O - E) performance deviates positively from the zero line.

Plotting, for each tactic, the cumulative values of the O curves on the abscissa and the corresponding (O - E) values on the ordinate, convex sets are obtained. These sets have been developed for problems 31A, 31B, 31C, 31D, 33A, 33B, 33C, 33D, 35A, 35B, 35C, 35D, for college and high school freshmen, as well as for problems 37A and 37B in college freshmen. The norms used for these convex sets were based on group performance. Convex sets for problems 26 and 41A were drawn for both student groups, using group and schemata norms.

The study of the convex sets corresponding to the training problems shows that college students tend to have higher ΣO values and less dispersion around the $\Sigma(O - E)$ values. That is, they are more consistent than high school freshmen. This adds further strength to the findings obtained with the other scoring methods.

A more detailed analysis was performed for problems 41A and 26, which were given at the post-testing session.

In Figures 14 to 21 inclusive we present the convex sets corresponding to problems 26 and 41A in both college and high school students using both group norms and schemata norms. All of these figures have been drawn to the same scale. The numbers in the figures correspond to the tactics followed. The sequence corresponding to each tactic is given at the bottom of each figure. Only tactics in the corners of the sets are numbered.

The discussion that follows is not all that can be done with these results. For the purposes of this report, we only want to call attention to some findings that are of interest. More work on this problem is now being actively performed.

When we compare the college and high school freshmen, using group norms, in problem 26 (Figures 14 and 16), we find that there is not a great difference between the convex sets for control and experimental, either for the college or the high school freshmen. In terms of the observed values (abscissa) the high school freshmen have higher values than the college freshmen. In terms of (O - E) the dispersion is greater for the high school freshmen than for the college freshmen.

The same type of comparison for problem 41A, (Figures 18 and 20) shows the same characteristics. Notice that in Figure 14, there is not a clear difference for the experimental and control subjects using problem 26, but the same comparison, when using problem 41A (Figure 18) shows that there is a wider dispersion of tactics for control in terms of observed as well as in terms of (O - E) values. This trend seems to also be discernible in Figure 20 for problem 41A in college students. The smaller height of the convex sets shown by the experimental subjects could be interpreted as indicating more agreement among themselves than in the case of the control subjects.

Since the vertical dimension indicates (O - E) values, where the expected are randomly obtained, this seems to indicate that in terms of group norms, college students depart less from random than high school freshmen. This is a surprising finding and seems to disagree with what has been said in the rest of this text.

When comparing high school and college freshmen using schemata norms, in problem 26 (Figures 15 and 17), we find that for the high school freshmen (Figure 15) the convex set corresponding to the experimental subjects shows higher observed values than the set for the control subjects, and that the same trend is found for the (O - E) values. Notice that the (O - E) values are higher for the college freshmen than for the high school freshmen.

For problem 41A (Figures 19 and 21) the same trend holds, but more clearly. That is, while in terms of observed values there is not much difference between control and experimental at the high school level (Figure 19), there is a clear differentiation in favor of the experimental subjects at the college level (Figure 21). Also the values for (O - E) are higher in college students and always greater for the experimental subjects.

It may be appropriate to point out again the fact that, in terms of group norms, the college students departed much less from randomness than the high school freshmen, but that, in terms of schemata norms, the findings are exactly the opposite. This suggests very strongly the inadequacy of group norms. It would require a "tour de force" to interpret these results otherwise, since all the previous evidence shows that college freshmen behave "better" than high school freshmen in the performance of the tests. The convex sets, as here discussed, bring this characteristic into a clearer focus.

Also, problem 41A turns out to be a better instrument than problem 26. Another point of interest relates to the corners of the convex sets and the tactics that define them. It had been suspected for a while that families of tactics can be described along the boundaries of the sets. This seems to be borne out in raw data and more clearly so for those sets based on the schemata than those based on group norms, though these latter ones also seem to follow the same trend.

By the way in which our sets have been presented, the dominant tactics should align themselves on the upper boundaries of the sets, and as we curve from top left to bottom right and then again to top left, a progressive change in the sequences corresponding to the tactics should be observed. Here, we shall not discuss points of maximum and minimum and mixed tactics, but it is clear that any tactic could be expressed as a linear function of any two or more tactics following on the same straight line.

For problem 41A, the schemata are as follows (the numbers represent questions):

1, 5, 6, 9, 3
1, 6, 9, 5, 3
1, 6, 9, 3, 5
14, 1, 6, 9, 5, 3
14, 1, 6, 9, 3, 5
14, 1, 5, 6, 9, 3

On these bases, and as described before, values were assigned to the questions. Looking at Figure 21, for experimental, we find that tactic 2 corresponds to the sequence 1, 5, 6, 9, 3; tactic 17 corresponds to the sequence 10, 6, 9; tactic 9 to sequence 3, 6, 5; tactic 5 to sequence 6, 3, 8, 13; and tactic 7 to sequence 1, 6, 3, 2, 13, 8, 11, 5, 9. By comparing these sequences we find that those in the upper left corner agree more closely with the schema, and that those along the lower boundaries agree less, with the exception of those approaching the top left corner.

Much work is needed to clarify some of the new findings that may be forthcoming using this approach. The reader is invited to peruse Figures 14 to 21, and interpret the tactics in each one of them, using the sequences given at the bottom of each figure. For problem 26, the schemata were:

1, 6, 7, 8, 9
1, 6, 9, 7, 8
1, 7, 8, 6, 9
1, 7, 8, 9, 6
1, 6, 7, 9, 8
1, 7, 9, 6, 8
1, 9, 6, 7, 8
1, 9, 7, 8, 6
1, 9, 7, 6, 8
1, 7, 6, 9, 8

The possibility of evaluating any tactic as a function of others does not seem, at the present, such a far away goal, and we expect to explore this possibility further.

In this report, because of lack of time, no explanation of the convex sets corresponding to questions not asked will be attempted.

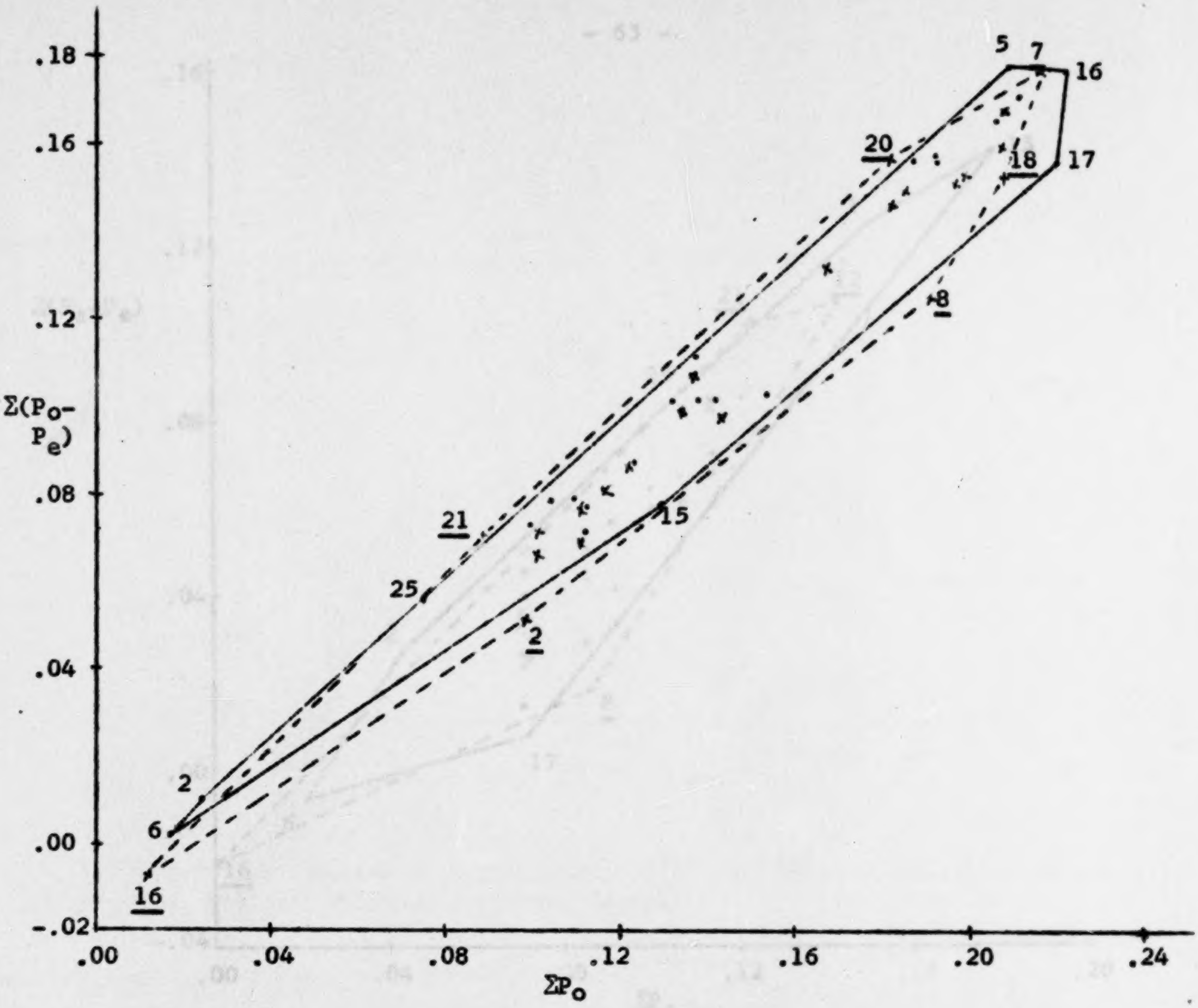


Figure 14. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 26 Based on Group Norms.

Control Tactics

7-1,2,3,4,7,8,9,6
20-1,2,3,4,7
21-1,6,7,10
16-12,10,5,11
2-1,9,7,2,3,4,12,11,10
8-1,2,6,3,7,8,4,9,13,14,5,12,11,10
18-1,2,3,4,9,13,7,8,6,14,12

Experimental Tactics

5-1,2,3,4,7,9,6
25-1,9,6,13
2-6,7,8
6-4,6,11
15-1,6,9,7,4,3,8,10,5,2
17-1,2,3,4,7,6,9,13,5,14,11,10,8
16-1,2,3,4,7,8,9,6,11

Note: In the above figure and the following convex sets an unbroken line depicts experimental performance and a broken line depicts control performance. Likewise in all the convex sets Ignatius refers to performance of high school subjects and Loyola refers to college subjects.

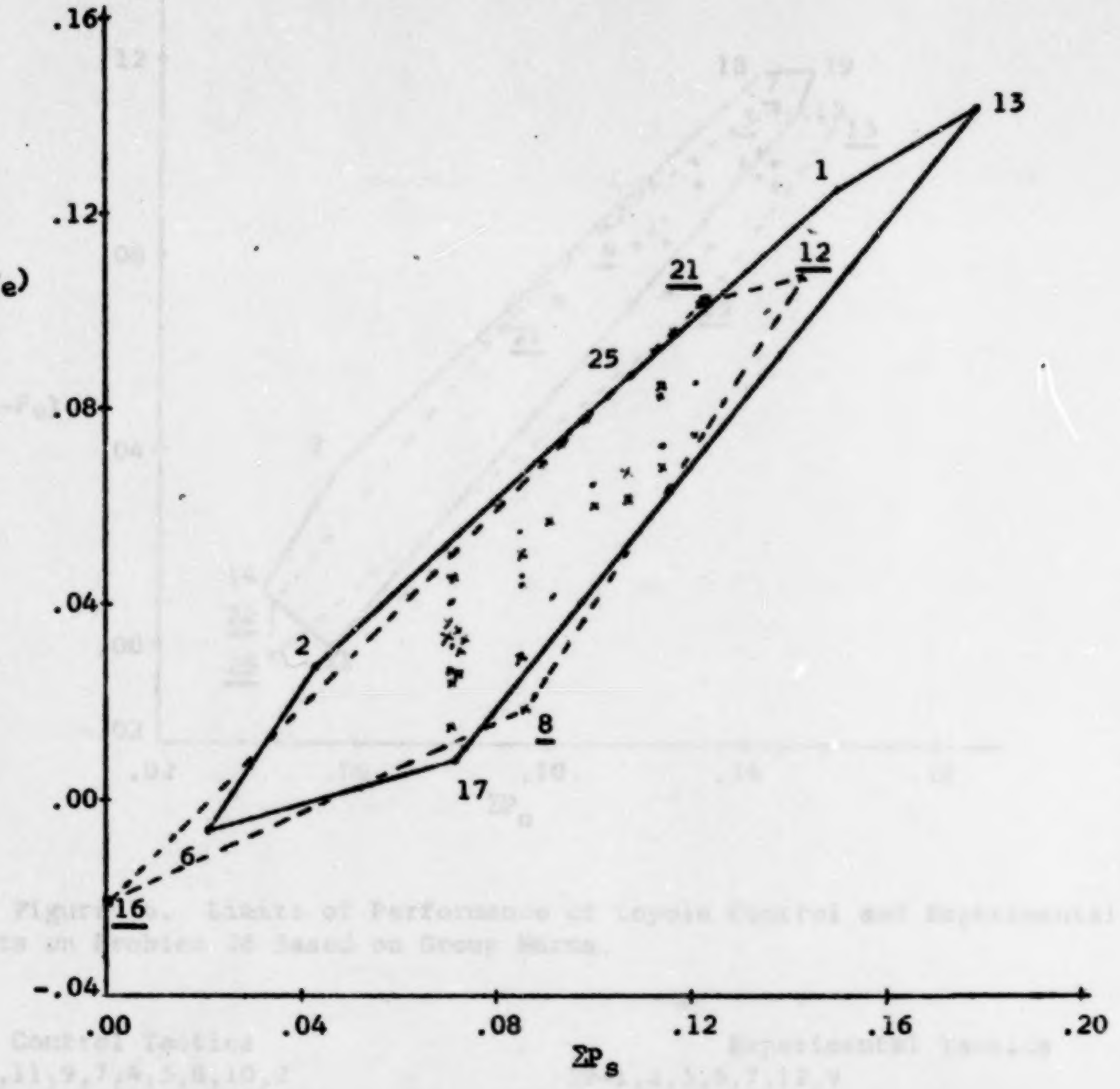


Figure 15. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 26 Based on Schemata Norms.

Control Tactics

12-1,2,6,7,8,9,5
21-1,6,7,10
16-12,10,5,11
8-1,2,6,3,7,8,4,9,13,14,5,12,11,10

Experimental Tactics

13-1,7,9,6,8,13,5
1-1,6,7,8,9
25-1,9,6,13
2-6,7,8
6-4,6,11
17-1,2,3,4,7,6,9,13,5,14,11,10,8

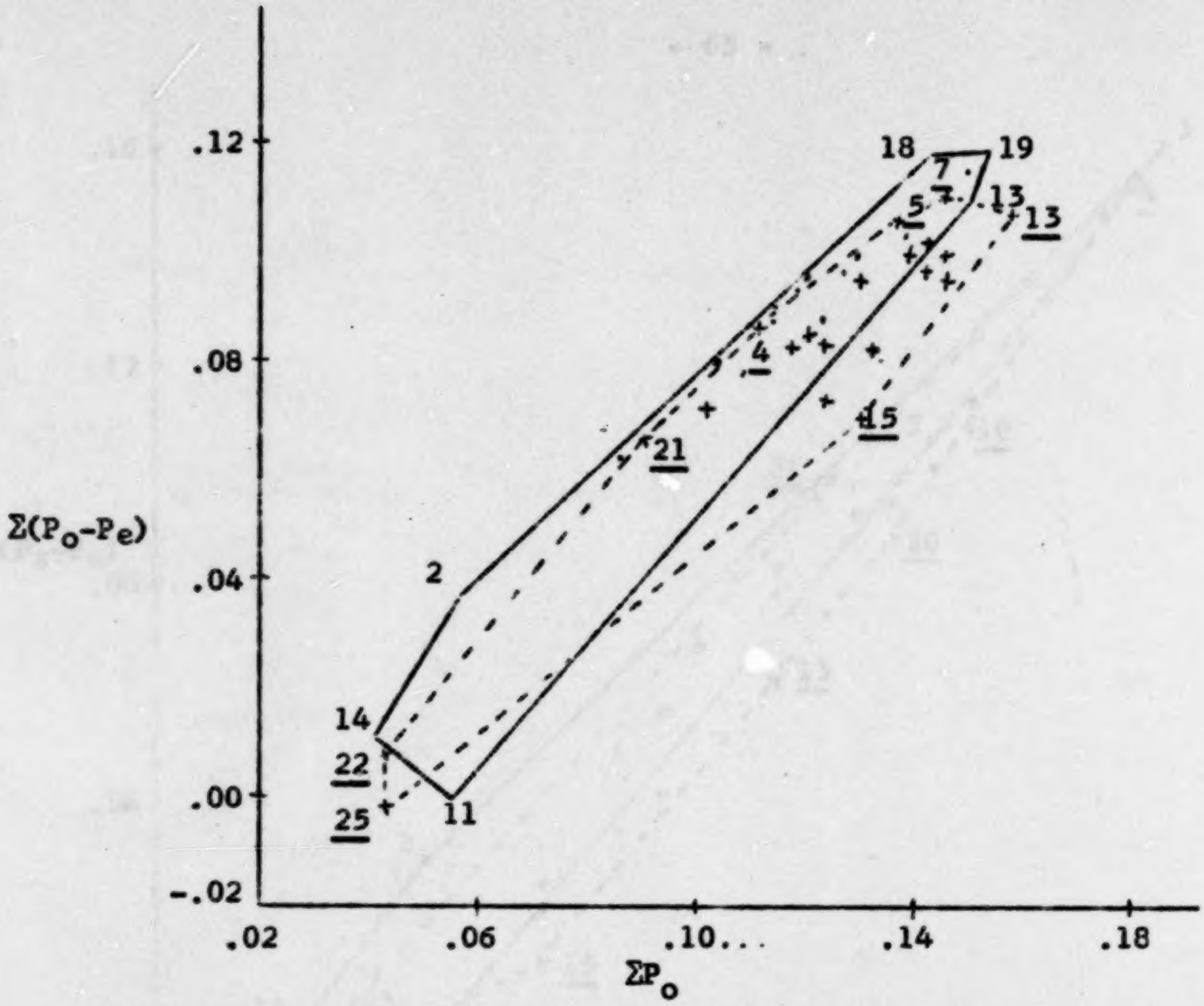


Figure 16. Limits of Performance of Loyola Control and Experimental Subjects on Problem 26 Based on Group Norms.

Control Tactics

13-1,6,11,9,7,4,3,8,10,2
7-1,2,3,6,7,9,14
5-1,6,3,9,7,8
4-1,7,9,6,8
21-1,9,8,3,6
22-3,7,1,2,4,6,10
25-13,14,7,8,1,12,6,5,3
15-1,10,11,12,2,3,4,6,5,9,7,8

Experimental Tactics

19-1,2,3,6,7,12,9
18-1,2,3,9,6
2-9,7,3,6
14-7,14,13,8,9,2
11-6,10,11,8,12,14,3,7,13,5,4,9
13-1,2,4,9,7,6,14,8

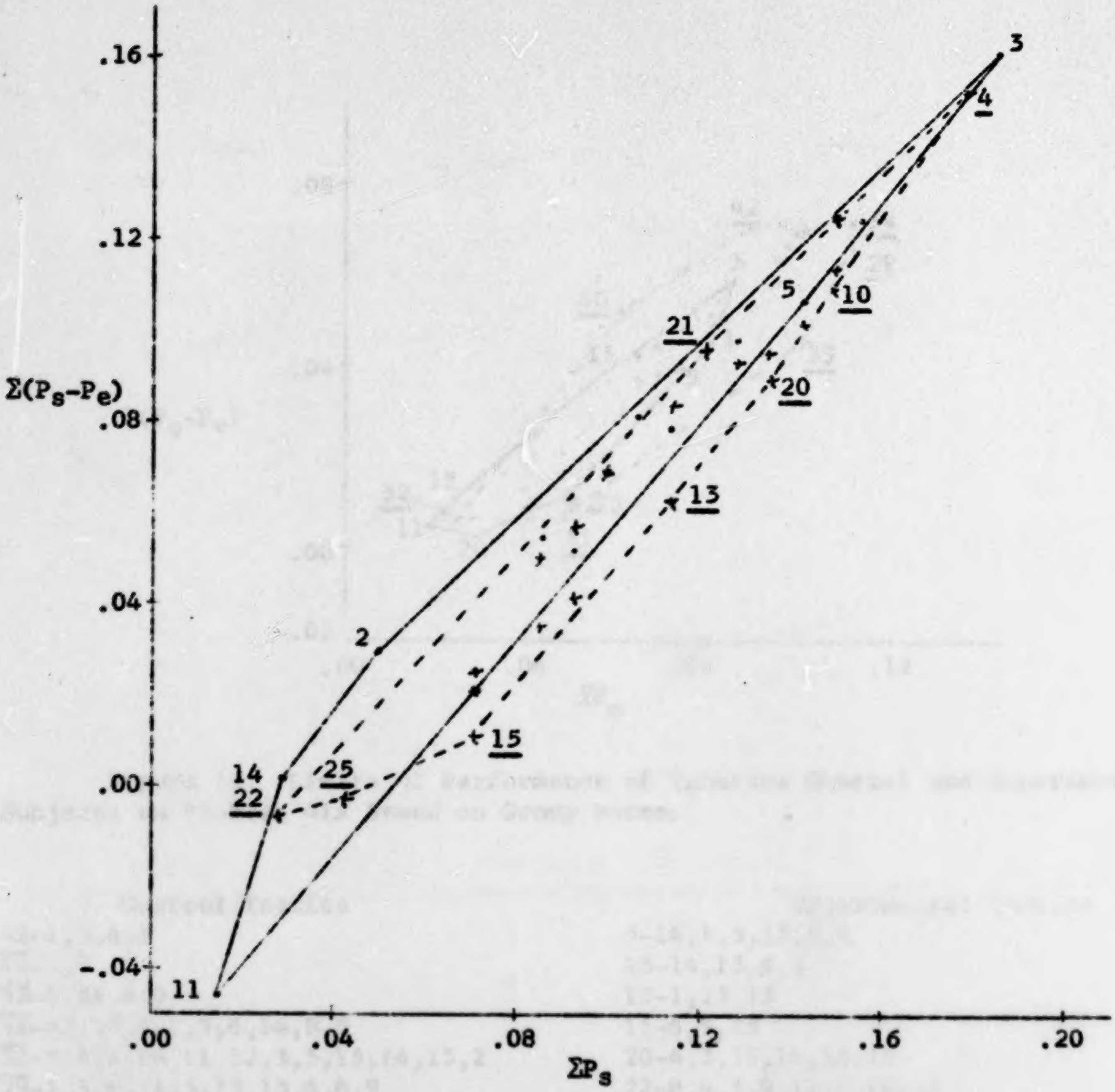


Figure 17. Limits of Performance of Loyola Control and Experimental Subjects on Problem 26 Based on Schemata Norms.

Control Tactics

4-1,7,9,6,8
21-1,9,8,3,6
22-3,7,1,2,4,6,10
25-13,14,7,8,1,12,6,5,3
15-1,2,3,4,6,12
13-1,6,11,9,7,4,3,8,10,2
20-1,6,7,4,9,3,10,8,5
10-1,6,2,7,8,11,4,9

Experimental Tactics

3-1,9,7,6,8
2-9,7,3,6
14-7,14,13,8,9,2
11-6,10,11,8,12,14,3,7,13,5,4,9
5-1,6,7,9,2,3,8

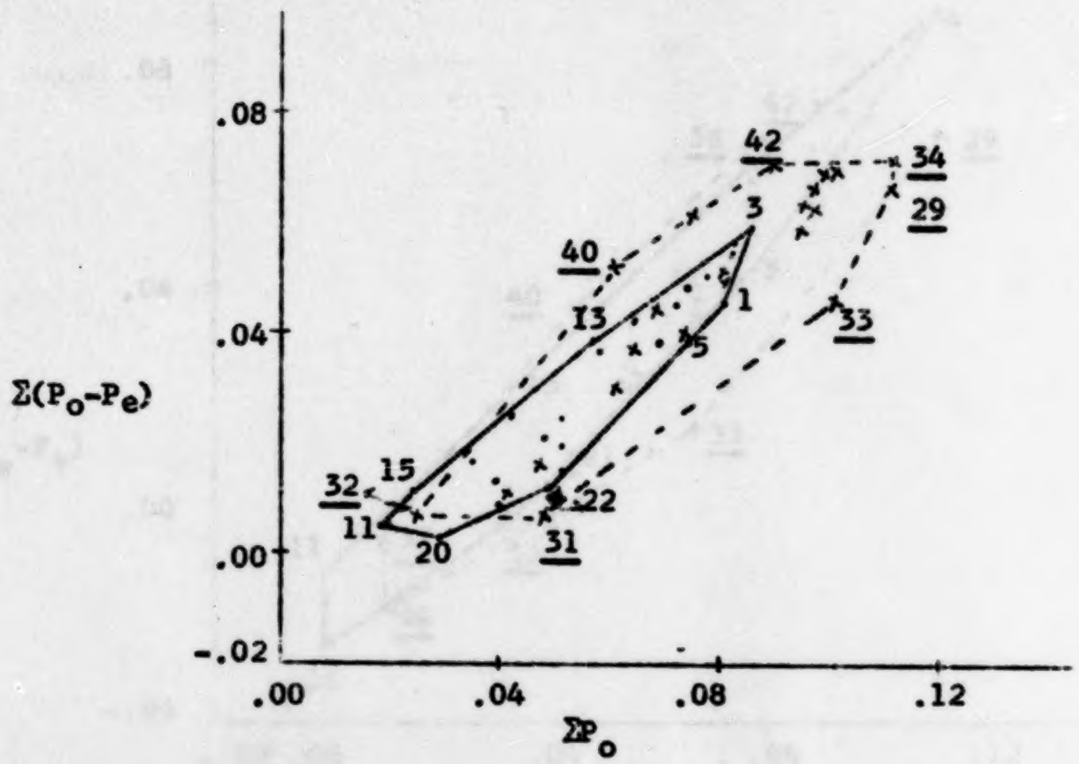


Figure 18. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 41A Based on Group Norms.

Control Tactics

42-1,5,6,3
40-1,5
32-5,14,4,3
31-15,13,4,1,3,6,14,8,5
33-1,4,6,10,11,12,3,5,15,14,13,2
29-1,5,6,11,3,15,13,4,8,9
34-1,11,14,2,13,6,9,3,5

Experimental Tactics

3-14,1,3,13,5,9
13-14,13,6,3
15-1,15,13
11-6,9,15
20-4,3,11,14,10,15
22-8,4,3,9,11,5,10,13
5-14,11,6,10,9,3,13,5
1-14,8,4,1,6,9,3,5

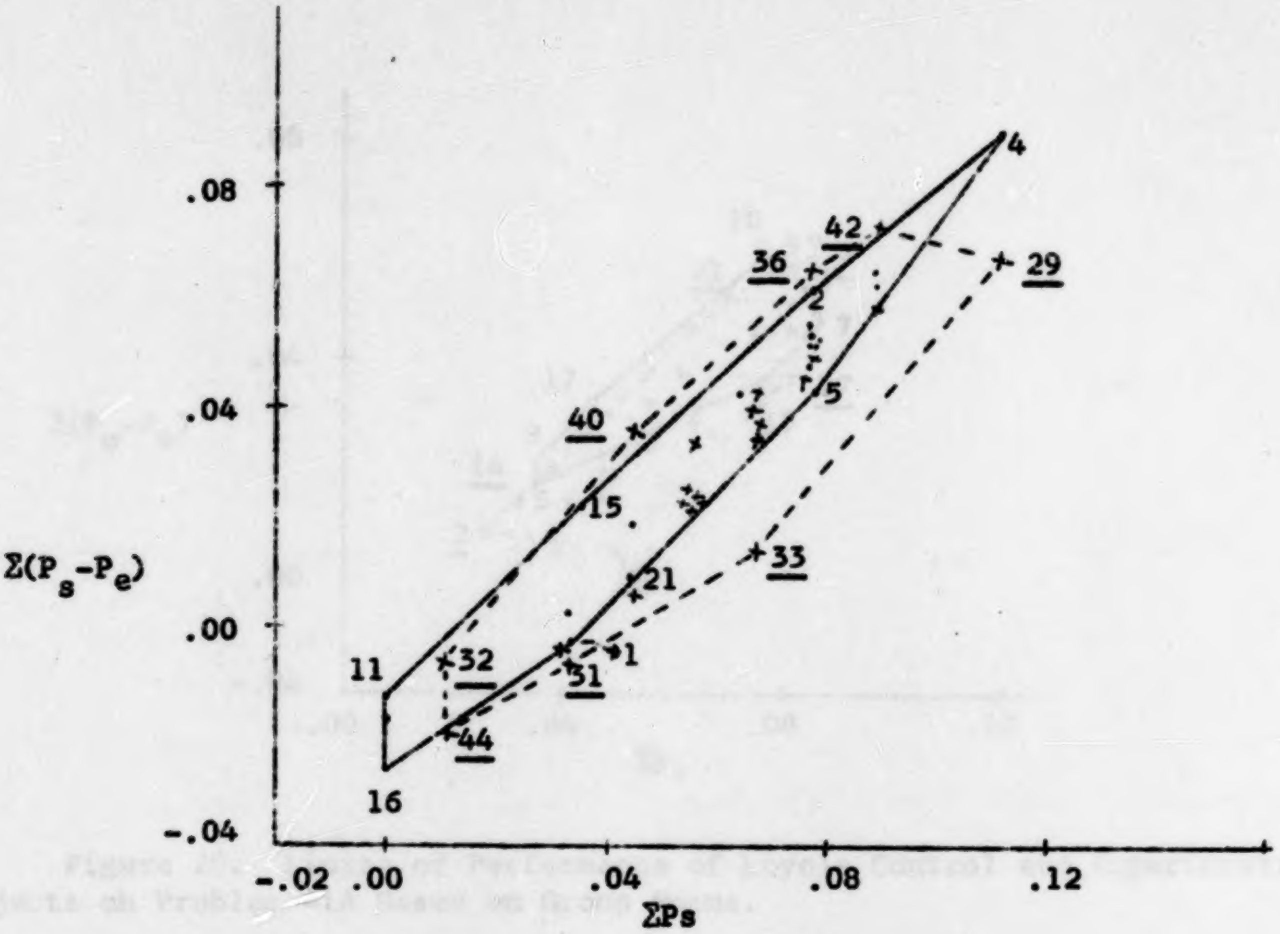


Figure 19. Limits of Performance of Ignatius Control and Experimental Subjects on Problem 41A Based on the Schemata Norms.

Control Tactics

42-1,5,6,3
36-1,5,6
40-1,5
32-5,14,4,3
44-6,11,3,15,13,5,10
31-15,13,4,1,3,6,14,8,5
33-1,4,6,10,11,12,3,5,15,14,13,2
29-1,5,6,11,3,15,13,4,8,9

Experimental Tactics

4-1,9,6,5,3
2-1,5,3,9
15-1,15, 13
11-6,9,15
16-6,3,13,10,8,11
1-14,8,4,1,6,9,3,5
21-12,1,11,5,2,6,3,9
5-14,11,6,10,9,3,13,5

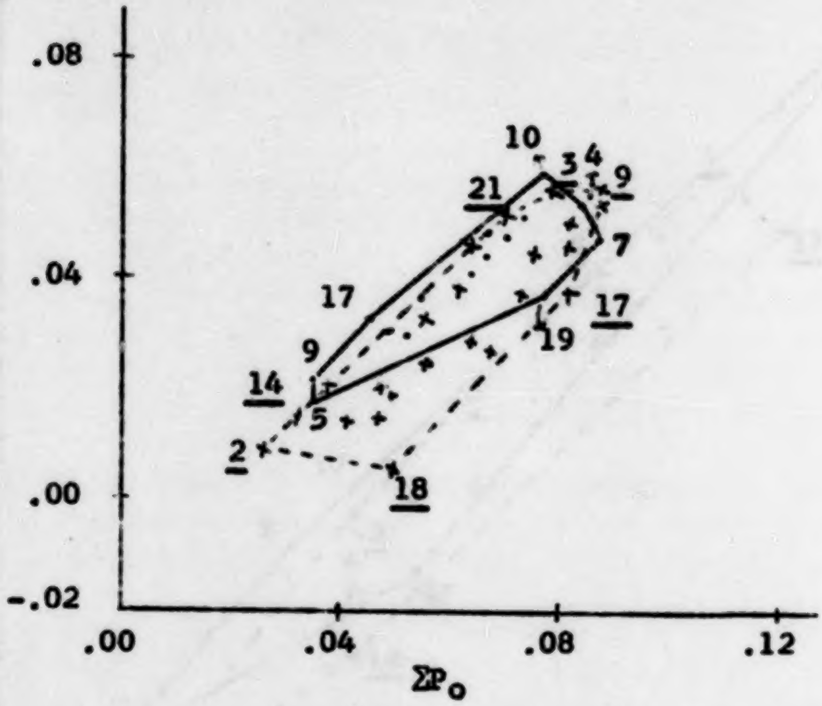


Figure 20. Limits of Performance of Loyola Control and Experimental Subjects on Problem 41A Based on Group Norms.

Control Tactics

9-1,6,9,3,13,11,5
3-1,6,9,3,5
21-1,6,5,3
14-7,5,6,9
2-6,3,5,9
18-2,5,6,15,12,13,10,11,4,3
17-14,1,6,10,8,3,11,2,13,5

Experimental Tactics

10-5,6,3,9
17-10,6,9
9-3,6,5
5-6,3,8,13
19-5,6,13,1,3,14,4,8,10
7-1,6,3,2,13,8,11,5,9
4-1,6,11,3,13,5,9

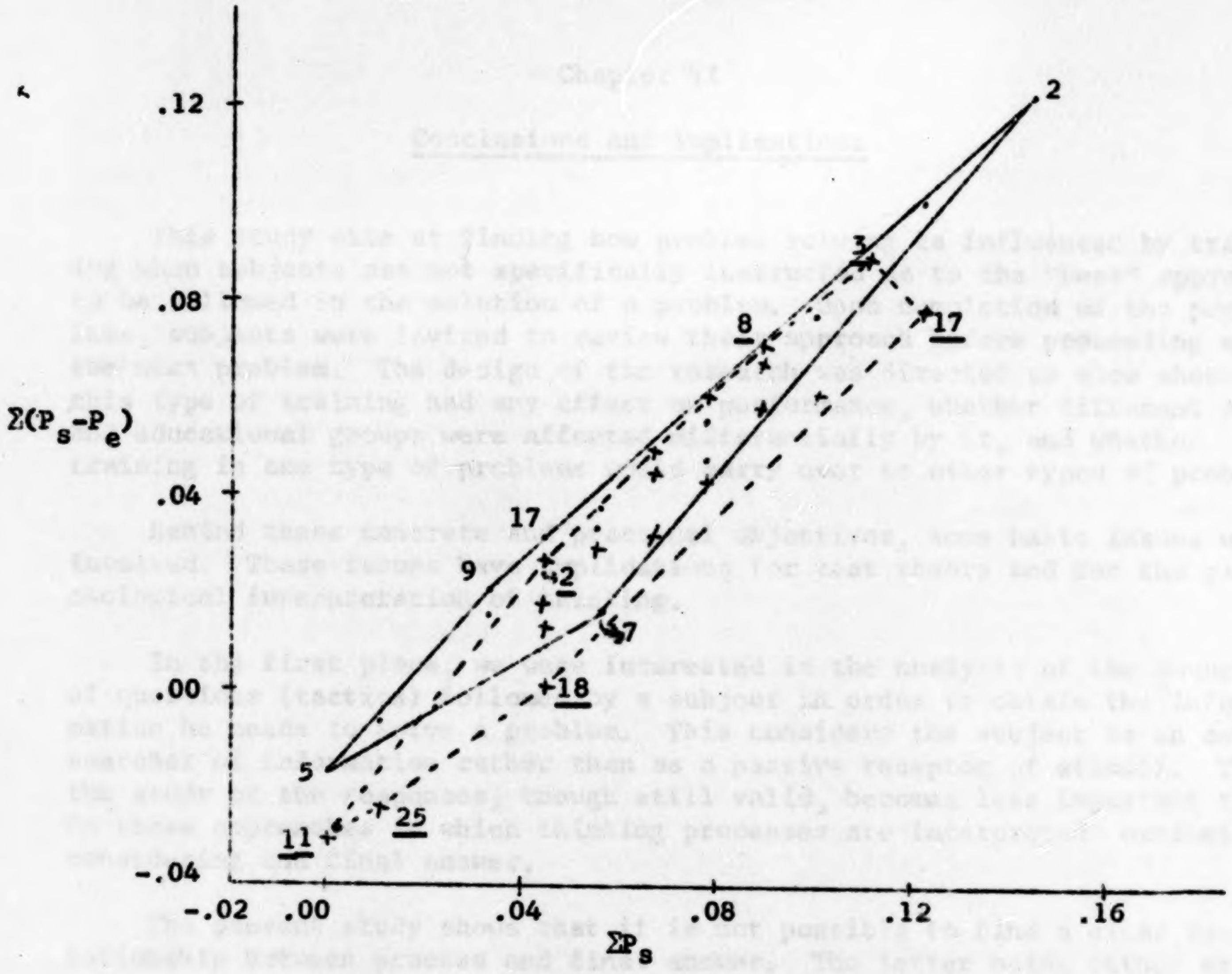


Figure 21. Limits of Performance of Loyola Control and Experimental Subjects on Problem 41A Based on Schemata Norms.

Control Tactics	Experimental Tactics
3-1,6,9,3,5	2-1,5,6,9,3
8-1,5,6,3	17-10,6,9
2-6,3,5,9	9-3,6,5
11-5,14,13,8,6,9,3	5-6,3,8,13
25-6,12,1,3,13,14,5,10	7-1,6,3,2,13,8,11,5,9
18-2,5,6,15,12,13,10,11,4,3	
17-14,1,6,10,8,3,11,2,13,5	

Chapter VI

Conclusions And Implications

This study aims at finding how problem solving is influenced by training when subjects are not specifically instructed as to the "best" approach to be followed in the solution of a problem. Upon completion of the problems, subjects were invited to review their approach before proceeding with the next problem. The design of the research was directed to show whether this type of training had any effect on performance, whether different age and educational groups were affected differentially by it, and whether training in one type of problems would carry over to other types of problems.

Behind these concrete and practical objectives, some basic issues were involved. These issues have implications for test theory and for the psychological interpretation of thinking.

In the first place, we were interested in the analysis of the sequence of questions (tactics) followed by a subject in order to obtain the information he needs to solve a problem. This considers the subject as an active searcher of information rather than as a passive receptor of stimuli. Thus, the study of the responses, though still valid, becomes less important than in those approaches in which thinking processes are interpreted exclusively considering the final answer.

The present study shows that it is not possible to find a clear relationship between process and final answer. The latter being either wrong or correct classifies subjects into two groups, while the processes, because of their variety are a more precise indication of the individual differences that may exist. As a matter of fact, many individual differences may be clouded because of this manner of procedure. Summarizing the results obtained, it seems clear that for a closer look at thinking processes, the solution of a problem may be a biased or unreliable indicator of them. Further, it would be difficult through the study of the answers to justify an experimental interpretation of processes. At most, this could be an inference attempting to explain what goes on in the subject's mind through the funnel of a preconceived hypothesis. This type of behavior may be endlessly repeated with varying degrees of success and failure but, in order to minimize the experimenter's bias, there is not yet a better substitute than to observe what happens by getting closer to the process. This is exactly what we tried to do in this research. It seems that this study strengthens our belief that the high points needed for a better understanding and explanation of the dynamics of thinking may not only be in the study of the final answers to a set of problems, but in the processes that lead to them. Originality in thinking, high talent and perhaps creativity, are not only a question of getting more answers correct, but also, of the way in which these answers are reached.

If these processes can be graphically presented, then the comparison between individuals is readily made, at least at the observational level. For this purpose, of the several approaches that were developed, two of them were used in this research. One of them referred to group norms and the other to norms developed following the schemata.

In relation to the schemata, their definition and control was one of the main issues involved. As a matter of fact, any thinking process may be conceived as reflecting the logical relationships within the problem itself and/or the development by the subject of a logical set of relationships made real by conventional codings. The logical sets are called schemata, the conventional codings are the contents used.

By controlling the schemata it can be investigated, by the sequence of questions asked, whether a subject understood the schema prepared by the experimenter or whether his logical set of relationships was different to the one embodied in the problem. A graded program of schemata was developed and each schema was realized using various types of content. These contents were arbitrarily assumed to represent material of varying degree of familiarity.

In terms of the schemata, sets of scoring norms were prepared. These norms are heavily influenced by the properties of the problem itself. (In the near future, we intend to analyze the dimensionality of the schemata. This will be, as it were, a control on our own subjectivity in defining schemata.).

When comparing performance curves developed in terms of group norms and in terms of schemata, it becomes clear that the former give a distorted view of each subject's process, by confusing, as it were, the behavior shown by each subject and the properties of the problems into a misleading set of values. This we have called extrinsic difficulty to differentiate it from the intrinsic difficulty, related to the schema.

Analyzing the plateaux (from the curves based on schemata norms), it is clear that both schemata and content contribute significantly to performance, and that training affects both differentially. This seems to follow the same pattern in both high school and college freshmen, though the latter are consistently superior to the former. Since plateaux can be interpreted as indicating defective planning, this may have important connotations in terms of training, and how this training may be expected to be effective in terms of increasing the subject's ability to deal with progressively more complicated sets of logical relationships and/or dealing with material of decreasing familiarity.

The design of the experiment permits testing the influence that training in problems, where the subjects generate their own questions (type b), has on those where the subjects choose questions from a given set (type a), and vice versa. Problems of type b consisted of drawings in which lines, areas and shapes were controlled by the experimenter.

The findings indicate a transfer of training more noticeable when subjects were trained in problems of type a. This result may be due to the fact that our understanding and control of problems of type a was better than for problems of type b. As a matter of fact, there is greater doubt concerning the schemata for problems of type b than for those of type a, with the addition that in the former a larger number of variables may be affecting performance. Nevertheless, in the final testing sessions, it was found that, whether subjects have been trained first in problems of type a or in problems of type b, the final result was the same, as judged in terms of the problems administered in the post-testing session.

There is a great temptation to evaluate "goodness" of performance by considering number of questions asked, under the assumption that those subjects who ask less questions are able to handle the information more precisely, that is, to draw more and better conclusions from less questions. But, of course, the opposite assumption may also be defended if it is assumed that the "best" subjects would like to have as much information as possible before reaching a solution. In this case more questions would indicate better ability. Early in our exploration of this area, we found that psychologists and educators were clearly divided on this issue. For instance, when a technique like the one used in this study was applied to the study of medical diagnostic processes, several instructors reported that the best students are those who ask as many questions as possible (Rimoldi, Haley, Fogliatto, 1962). However, it was also mentioned that one of the shortcomings often observed in students was the fact that they did not know how to combine important information in their diagnostic performances,

These differences in opinion seem to be related to a lack of a clear definition of educational aims. If the purpose of an educational program is to have subjects acquire as much information as possible, then a great number of questions, including those that may be only slightly relevant to the problem at hand, would be an indication of "good" performance. But, if the aim of an educational program is to allow people to handle and interpret information more thoroughly, then accumulation of information per se may not be the most desirable aim. As a matter of fact, it is our feeling that a point of diminishing returns is reached in problem solving performances when too much information that is not completely pertinent is manipulated, and that often a premium is placed on accumulating information rather than on using it in the most effective way. Indeed, the most effective way may be a large number of ways, each one of them highly personal. If this is properly considered, then originality in thinking can not be evaluated in terms of group performances that by necessity will punish some of those that deviate from a group norm.

All our experience on this matter tends to indicate that with increased experience and knowledge, subjects ask less questions and eliminate those that are irrelevant. In this study, the same trend was found, that is, college students on the whole ask for less questions than high school students and experimental subjects also ask for less questions than control subjects.

If an interpretation of these findings is permitted, a possible explanation could be that with increased knowledge and increased experinece, more discrimination is made between what is relevant and what is not and more attention is paid to the understanding of the problem and to the planning of the tactic that the subject considers is most likely to succeed. This interpretation draws together, on the whole, information coming from this study.

This investigation seems to indicate that tactics and number of questions asked are not clearly related to number of correct solutions to the problems administered. This same finding has been found in other studies. Here again, the question of evaluation depends on a statement of aims. It has been found (Rimoldi, Meyer, unpublished) that when old persons of high educational level are administered some of the usual IQ tests, their scores, though lower in general than those of younger subjects, are high in terms of the norms. Nevertheless, in instruments that are less dependent on everyday experience, like Raven's Progressive Matrices Test and in our type of problems, their performances are very poor. As a matter of fact, a graphic way of expression would be to say that they "refuse" to think. Nevertheless, as implied by their IQ's, they answer correctly many of the items of the test.

We shall not discuss here all the implications of these observations, but only point to the fact that, familiarity or good "conditioning" to the requirements of the usual environment may not necessarily be an indication of high thinking ability. In some situations, this may be a very important issue, for instance, for the appraisal of high talent. It is almost an aphorism that in scientific research, what matters is not exclusively to obtain clear-cut solutions to problems, but rather to plan the attack that will bring about more fruitful results. As a matter of fact, the history of science shows that those scientists who have distinguished themselves have been able to get into the riddle of a problem by asking pertinent questions that may or may not give the "correct" solution to the issue at hand. Thus, if our design and instruments are considered to be directed towards the study of thinking, it is not surprising to find out that "correctness" of the final answers may not be a good indicator of them.

The study of the ellipsoids shows that the experimental subjects agree among themselves better than the controls. Also, that college students are, on the whole, better than high school students. The norms used here are based on group performances, and thus cloud individual differences. The importance of this finding is then in showing how training may increase agreement.

A large number of mathematical and statistical methods were either adapted or invented for the purposes of this research. It should be obvious that the usual techniques could not always be applied to our data. Some of these are not yet completely developed, others lack, as yet, an appropriate test of significance, and still others are in an embryonic state. These techniques have been described in the body of the text or in already published studies.

In relation to improvement of school grades in those subjects that were trained with the problems here described, nothing definite can be reported, though it seems clear that experimental subjects gained more than control subjects in their average grades (mid and final grades).

In this report we have not presented the complete analysis of our data because of lack of time and help. This analysis we expect to complete in the future.

The implications of this research in areas other than education extend to the study of changes in thinking processes with age, in abnormal conditions, and under the effects of pharmacological agents, as well as to the early detection of high talent in specific fields, such as mathematics.

References

Bloom, B. S. and Broder, L. G., Problem Solving Processes of College Students, Chicago: University of Chicago Press, 1950.

Bruner, J. S., The Process of Education, Cambridge: Harvard University Press, 1960.

Bruner, J. S., Goodnow, J. J., Austin, G. A., A Study of Thinking. John Wiley & Sons, New York, 1960.

Bryan, G. L., The Automasts: An Automatically Recording Test of Electronic Trouble Shooting. Los Angeles: University of Southern California, Report No. 11, 1954.

Buswell, G. T., Kersh, B. Y., Patterns of Thinking in Solving Problems. University of California Publications in Education, University of California Press, Berkeley and Los Angeles, 12, 2, 1956.

Duncker, K., On Problem Solving. Psychological Monographs, 58, 5, 1945.

Glaser, R., Damrin, D. E., Gardner, F. M., The Tab Item Technique for measurement of proficiency in diagnostic problem solving tasks. Educational and Psychological Measurement, XIV, 2, 1954.

Goodman, L. A., Kolmogorov-Smirnov Tests for psychological research. Psychological Bulletin, 51, 2, 1954.

Gunn, H. E., Appraisal of personality parameters in terms of processes, 1961. (unpublished Ph.D. dissertation).

John, E. R., Contributions to the study of the problem solving process. Psychological Monographs, 71, 18, 1957.

John, E. R., and Rimoldi, H. J. A., Sequential observation of complex reasoning. The American Psychologist, 470, 1955.

Johnson, D. M., Formulation and reformulation of figure concepts. American Journal of Psychology, LXXIV, 1961

Mohrbacher, J. W., An analysis of the interdisciplinary evaluation of organic pathology in a child guidance setting, 1960. (unpublished Ph.D. dissertation).

Rimoldi, H. J. A., A technique for the study of problem solving. Educational and Psychological Measurement, 15, 4, 1955.

Rimoldi, H. J. A., L'Étude des processus psychologiques. *Travail Humain*, France, 1961.

Rimoldi, H. J. A., Devane, J. R., Training in problem solving. Chicago: Loyola University, Loyola Psychometric Laboratory, 1961. (publication No. 21).

Rimoldi, H. J. A., Devane, J. R., Grib, T. F., Testing Skills in Medical Diagnosis. Chicago: Loyola University, Loyola Psychometric Laboratory, 1958. (publication No. 2).

Rimoldi, H. J. A., Devane, J. R., Haley, J. V., Characterization of processes. *Educational and Psychological Measurement*, XXI, 2, 1961.

Rimoldi, H. J. A., Grib, T. F., Pattern Analysis. *British Journal of Statistical Psychology*, November, 1960a.

Rimoldi, H. J. A., Grib, T. F., Some properties and applications of pattern analysis. Chicago: Loyola University, Loyola Psychometric Laboratory, 1960b. (publication No. 14).

Rimoldi, H. J. A., Haley, J. V., Sequential evaluation of problem solving processes. Chicago: Loyola University, Loyola Psychometric Laboratory, 1962a. (publication No. 22).

Rimoldi, H. J. A., Haley, J. V., Determining significance levels in pattern analysis. *Psychological Reports*. 10, 500, 1962b.

Rimoldi, H. J. A., Haley, J. V., Fogliatto, H., The Test of Diagnostic Skills. Chicago: Loyola University, Loyola Psychometric Laboratory, 1962. (publication No. 25).

Rimoldi, H. J. A., Haley, J. V., Fogliatto, H., Erdmann, J. B., Program for the study of thinking. (unpublished, 1962).

Rimoldi, H. J. A., Haley, J. V., Majewska, Sister M. Canisia, C.S.F.N., Decision Processes in Mathematical Thinking. (to be published, 1963).

Rimoldi, H. J. A., Meyer, R. A., Meyer, M. L., Fogliatto, H., Psychobiological mechanisms in complex mental processes and their changes with age. Chicago: Loyola University, Loyola Psychometric Laboratory, 1962. (publication No. 24).

Tabor, A. Process analysis of Rorschach interpretation, 1959. (unpublished Ph.D. dissertation).

Tate, M. W., Stanier, B., Harootunian, B., *Differences between Good and Poor Problem Solvers*. School of Education, University of Pennsylvania, Philadelphia, Pa., 1959.

Wertheimer, M., *Productive Thinking*. Harper & Brothers, New York, 1959.

Name__________________________School_________________________Date____________

THOUGHT PROBLEMS, PART I

Directions: On the following pages there are 20 thought problems. Some are hard, but you can solve most of them if you read and re-read carefully, think straight, and take enough time. If you find some extremely hard, go on to the next one and return to the hard ones later.

In the problems which require figuring on paper, please use the space provided and the margin of the page. Be sure to show all of your figures so that credit can be given for correct steps even though your answer may be wrong.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. In our school two-thirds of the boys play football and two-thirds play basketball. Of the following statements, which is necessarily true?

   Some boys play neither football nor basketball.
   Some boys play both football and basketball.

2. In a certain school the positions of principal, assistant principal, and coach were held by Green, Burns, and Smith, though not necessarily in that order. The principal, who was an only child, earns the most. Burns, who married Green's sister, earns less than the coach.

   What position is held by Green? ______________________
   What position is held by Burns? ______________________
   What position is held by Smith? ______________________

3. When you enter my house you will find a window on your right in the side wall of the entry. When the sun sets it shines straight through this window on the opposite wall. What direction are you facing when you stand in the doorway and look accross the street?

4. Suppose that the Yankees lead the league and that the Red Sox are fifth, while the White Sox are midway between them. If the Indians are ahead of the Red Sox and the Tigers immediately behind the White Sox, which team is in second place?

5. In a farmer's papers, a receipt was found: 24 dozen eggs . . . . $_2.4_. The first and last digit of the amount had been blotted out.

   What are the two missing digits?

   What was the price of a dozen eggs?

6. I started from the school and walked 100 yards; I turned to the right and walked 50 yards; I turned to the right again and walked 100 yards. How far from the school am I?

7. There are six push buttons lettered A, B, C, D, E, F. Two of the buttons will ring a bell, four will not. The buttons can be pushed in combinations only of three. It is known that:

   Combination A, B, C rings the bell.
   Combination B, C, E does not ring the bell.
   Combination C, D, E rings the bell.
   Combination B, C, D rings the bell.
   Combination D, E, F rings the bell.

   Which two buttons are connected with the bell?

8. My birthday is on December 27, and I am just four days older than Tom. This year Christmas day comes on Tuesday. On what day of the week is Tom's birthday?

9. A pound of meat should roast for half an hour; two pounds of meat should roast for three-quarters of an hour; three pounds of meat should roast for one hour; eight pounds of meat should roast for two hours and a quarter. If you were writing a cookbook, how would you state the rule by which a person could tell from the weight of a piece of meat how long it should roast?

10. Three playing cards are placed in a row, face down. To the right of a King, there are one or two Queens. To the left of a Queen, there are one or two Queens. To the left of a heart, there are one or two spades. To the right of a spade, there are one or two spades. One of the cards is the Queen of hearts. What are the other two?

11. The witness said, "I heard my clock strike yesterday, about ten minutes before the shot was fired. I did not count the chimes, but I'm sure it struck more than once and that it struck an odd number. The witness had been out all morning and her clock stopped at 5 minutes to 5 the same afternoon. When do you think the shot was fired?

12. If the A's have a bigger army than B's, we ought first either to fight the B's or attack the C's by sea, but not attack the A's. If the A's army is smaller, we should attack the A's first. If the C's have a bigger navy than we, we ought to either fight the B's or the A's, but not the C's. If C's navy is smaller, we should first attack the C's by sea. The sizes of armies and navies are -

	Men	Ships
A	7,000,000	300
B	5,000,000	400
C	4,000,000	500
Ourselves	6,000,000	200

Whom should we attack first?

13. A cube is 3 inches on each side. It is painted red, then sliced up into one-inch cubes.

How many one-inch cubes will there be? ______________

How many one-inch cubes will have 3 red sides? ______________

How many one-inch cubes will have 2 red sides? ______________

How many one-inch cubes will have 1 red side? ______________

How many one-inch cubes will have no red side? ______________

14. Bill came home with a new bicycle. There is clay on his feet and soot on his forehead. The only places he can have been are West Philadelphia, North Philadelphia, South Philadelphia, and Camden. He has not had time to go to more than one of these. There are bicycle shops only in North Philadelphia, West Philadelphia, and Camden. There is soot in the air only in North Philadelphia, South Philadelphia, and Camden. The streets are not being repaired in North Philadelphia, but they may be in other places. Where had Bill been?

15. In numbering the pages of a book a printer used 111 digits. (The digits are 0,1,2,3,4,5,6,7,8 and 9.) How many pages has the book?

16. There are six push buttons lettered D, E, F, G, H, I. Two of the buttons acting together ring a bell. The buttons can only be pushed in combinations of three. It is known that:
    Combination DEF does not ring the bell.
    Combination EFG does not ring the bell.
    Combination FGH rings the bell.
    Combination GHI does not ring the bell.
    Which two buttons acting together ring the bell?

17. What idea can you draw from the following facts? Iron nails will not float in a pool; a cup of pure gold dust weighs nearly twenty times as much as a cup of water the same size; if you drop a dime or penny into a puddle it will sink to the bottom; a leaden weight will drop to the bottom of the ocean; a cubic inch (about a tablespoon) of water weighs less than half an ounce, while a cubic inch of brass weighs over two ounces. Sum up all these facts into one short statement:
    Most____________ are______________ ______ _______ _________.

18. Where the climate is hot, bananas and rubber trees will grow. Strawberries and potatoes will grow where it is cool. Strawberries and rubber trees require plenty of moisture; potatoes and bananas need fairly dry regions. Near the Amazon river it is very hot and very damp. Which of the above grows there?

19. A broker bought 5 shares of stock at $10 each which he sold at $8 each. But he sold at $3 each some shares he had bought at $2 each. If his profit was $15, how many shares did he sell at $3?

20. On a six-day vacation, John drank twice as much coke each day as he did the day before. On the 6th day he drank a quart. On what day did he drink a pint?

Name________________________School ____________________________Date__________

THOUGHT PROBLEMS, PART II

Directions: On the following pages there are 20 thought problems. Some are hard, but you can solve most of them if you read and re-read carefully, think straight, and take enough time. If you find some extremely hard, go on to the next one and return to the hard ones later.

In the problems which require figuring on paper, please use the spaces provided and the margin of the page. Be sure to show all of your figures so that credit can be given for correct steps even though your answer may be wrong.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

21. Some children were asked: "Why are towns nearly always more unhealthy than the country?" They gave the following replies: (Underline the best two replies.)

"Some country places are by the seaside."
"There are more doctors in the town."
"The smoke and fumes in the city make the air less pure."
"People in the country sleep more."
"Diseases spread more where people are crowded together."

22. The doctor thinks Jack has caught some disease. If he has a rash, it is probably chicken-pox, measles, or scarlet fever. If he has been ailing with a cold or cough, he may develop whooping-cough, measles, or mumps. He has been sneezing and coughing for some days, and now spots are appearing on his face and arms. What disease does Jack probably have?

23. John has 7 pockets and 20 dimes. He wants to put his dimes into his pockets so that each pocket contains a different number of dimes.

Can he do so?

Explain your answer.

24. Helen walked four blocks west on Elm Street. She turned left and walked two blocks, turned right and walked two more blocks, and then turned right again and walked one block. Which direction was Helen walking in the last block?

25. There are five books in a pile. The green one is directly under the blue one and is above the yellow one. The red one is above the brown one, but not next to it. The brown book is directly under the green book. Which book is on top?

26. There are a number of bacteria in a flask at 9 o'clock. There were half as many at 8 o'clock; there will be twice as many at 10 o'clock. At this rate the flask will be full at 4 o'clock. At what time will it be half full?

27. After eating at the company cafeteria, a number of workers got sick. Each worker had selected three items of food from the five: ham sandwich, cole slaw, rice pudding, milk, cocoa. The company nurse interviewed the workers and found that those who had eaten items ham sandwich, cole slaw, and milk got sick, as did those who had eaten cole slaw, rice pudding, and milk. However, those who had eaten ham sandwich, cole slaw, and cocoa didn't get sick. What two foods may have caused the sickness?

28. Bentonville is 75 miles east of Austin. Centerburg is 25 miles north of Bentonville. Dearborn is 75 miles west of Centerburg. How far is Dearborn from Austin?

29. Captain Watts and his son James have been found shot -- the father in the chest and the son in the back. Both clearly died instantaneously. A gun fired close to a person -- as, for example, when a man shoots himself -- will blacken and even burn the skin or clothes; fired from a greater distance it will leave no such mark. The two bodies were found near the middle of a large hall used as a rifle range. Its floor is covered with damp sand which shows every footprint distinctly. Inside the room there are two pairs of footprints only. A third man standing just outside the door or window could aim at any part of the room, but the pavement outside would show no footprints. Under Captain Watts' body was found a gun; no such weapon was found near James. In each case the coat, where the bullet entered, was blackened with gunpowder, and the cloth a little singed. Captain Watts was devoted to his son, and would have died rather than harm him purposely; hence it is impossible to suppose that he killed him deliberately, even in self-defense. But some think that James secretly disliked his father, and hoped to inherit his fortune at his death.

Was Captain Watts' death due to murder, accident, or suicide?

Was James' death due to murder, accident, or suicide?

30. The winning horse in a race finished at 4:31 P.M. He was six lengths ahead of the second horse, which finished nine lengths ahead of the third horse. The third horse ran the race in 2 minutes and 3 seconds. In the last quarter of the race, each horse was traveling one length in 1/5 second.

How long did it take the winning horse to win the race?

At what time did the race start?

31. During a class play one of the stagehands has to change the spotlight from red to blue. There are six switches that can be pulled. They are numbered 1, 2, 3, 4, 5 and 6. Two of the switches will turn on the blue light; the others will not. The stagehand knows that -----

1, 2 and 3 pulled together will make the light blue.
4, 5 and 6 pulled together will make the light blue.
2, 3 and 5 pulled together will make the light blue.
3, 4 and 6 pulled together will not make the light blue.
2, 5 and 6 pulled together will make the light blue.

Which two switches will turn the light blue?

32. A fireman stood on the middle rung of a ladder, directing water into a burning building. As the smoke lessened, he stepped up three rungs and continued his work from that point. A sudden flare-up forced him to go down five rungs. Later he climbed up seven rungs and worked there until the fire was out. Then he climbed the remaining six rungs and entered the building. Write down the number of rungs in the ladder.

33. I am at cross roads. I have come from the South and want to go to Philadelphia. The road to the right leads somewhere else. The road straight ahead leads only to a farm. In which direction is Philadelphia?

Circle the right answer: North, East, South, West.

34. Four boys, one of whom was known to have broken a window, were questioned by the principal. Each made a statement: Dave: "Tom did it." Tom: "Bill did it." Jack: "I didn't do it." Bill: "Tom lied when he said I did it." If only one of the four statements is false, who is the guilty boy?

35. George is running his little brother's train set. He wants to switch the engines onto a different track. There are six colored pushbuttons on the control panel - red, yellow, orange, green, blue, and purple. Two of the pushbuttons acting together will switch the train to a different track. George knows that -

Pushbuttons Orange, Green, and Purple acting together will not switch tracks.
Pushbuttons Red, Yellow, and Blue acting together will not switch tracks.
Pushbuttons Red, Yellow, and Orange acting together will switch tracks.
Pushbuttons Yellow, Orange, and Green acting together will not switch tracks.

Which two pushbuttons acting together will switch the tracks?

36. Dave bought the following Christmas presents: a pipe, some music, a carton of cigarettes, a bracelet, a toy engine, a bat, a book, a doll, a walking stick, and an umbrella. His brother is 18; he does not smoke, nor play baseball, nor play the piano. Dave wants to give the walking stick to his father and the umbrella to his mother. Which of the presents shall he give to his brother?

37. Three boxes are piled on top of each other. All are of the same size, 3 by 4 by 5 feet. The lower box rests on its 3 by 5 side, the second box on its 4 by 5 side, and top box on its 3 by 4 side. What is the height of the pile?

38. A cube is 3 inches on each side. It is painted white on two opposite sides; red on two opposite sides; and blue on two opposite sides. It is then sliced up into one-inch cubes.

How many one-inch cubes will there be? ________
How many will show the colors red, white, and blue? ________
How many will show white only? ________
How many will show red only? ________
How many will show blue only? ________
How many will show two colors only? ________
How many will show no colors? ________

39. A farmer sold 5 turkeys at $10 each, although it had cost him $12 to raise them. At the same time he sold at $2 each some chickens in which he had invested only $1 each. If his profit was $10, how many chickens did he sell for $2 each?

40. In cold, damp climates, root crops like potatoes and turnips grow best. In temperate climates there are many pastures, and oats and barley grow well. In subtropical climates wheat, olives, and vines grow well. In tropical climates date-palms and rice grow best. The ancient Verberites lived largely on bread, with oil instead of butter; they had wine to drink and raisins for fruit. Which climate do you think they had?

Problem 1

Instructions and Corresponding Questions and Answers

Figure I is composed of an overlapping rectangle, triangle, and circle, forming the areas A through E. You will notice that an area is any part of the figure that does not have a line through it. B, C, and D, for example, are separate areas, even though they are part of the same rectangle.

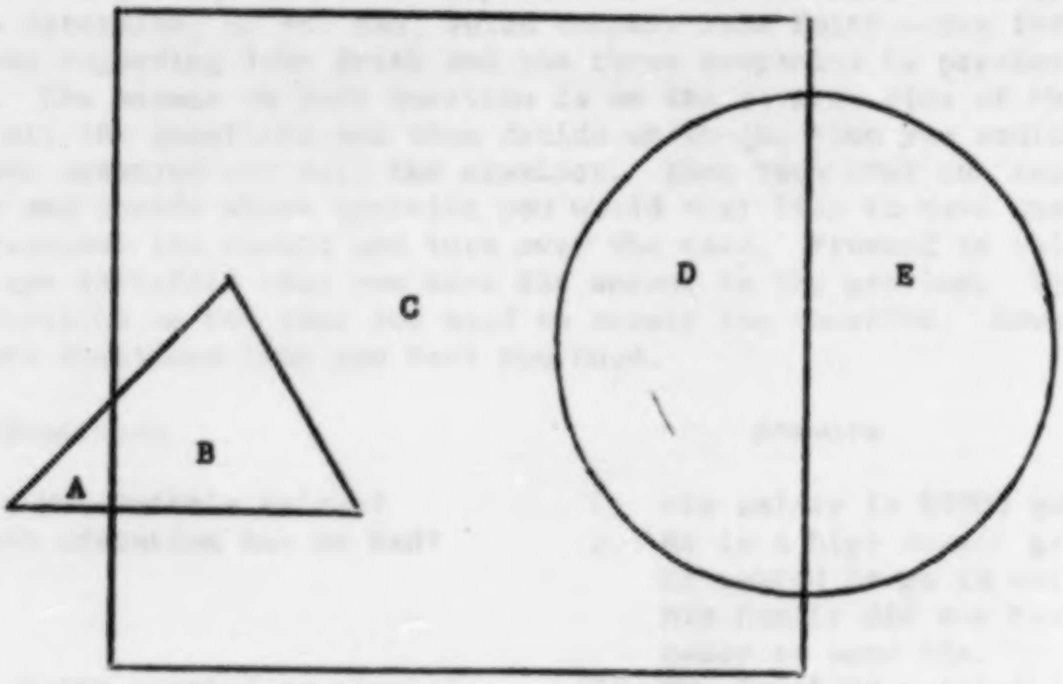
One of the areas has been picked at random. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from the other areas. You may discover these facts by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide on the first question you would like to have answered and write its number on the page provided. Then withdraw the card from the folder, and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number and withdraw the card. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may draw as many cards as you need to find the correct area, but don't draw any more than you need.

Questions	Answers
1. Is it the smallest area?	1. No, it is not the smallest area.
2. Is it the largest area?	2. No, it is not the largest area.
3. Is it outside the rectangle?	3. No, it is not outside the rectangle.
4. Is it within the rectangle?	4. Yes, it is within the rectangle.
5. Are all sides straight?	5. No, all sides are not straight.
6. Is it in two geometric figures?	6. Yes, it is within two geometric figures.
7. Does it have at least three straight sides?	7. No, it does not have at least three straight sides.
8. Is one border curved?	8. Yes, one border is curved.
9. Is it in a triangle?	9. No, it is not in a triangle.

Solution: The pre-selected area is D.

Appendix I



Problem 1

Appendix 1

Problem 19

Instructions and Corresponding Questions and Answers

John Smith lives in a small city that has only three companies. These are the Grant Co., the Williams Corporation, and the Gibbons Metals. Your task is to determine, if you can, which company John Smith works for. A set of questions regarding John Smith and the three companies is presented on the cards. The answer to each question is on the reverse side of the card. Look over all the questions and then decide which question you would first like to have answered and tell the examiner. Then turn over the card; read the answer and decide which question you would next like to have answered. Tell the examiner its number and turn over the card. Proceed in this way until you are satisfied that you have the answer to the problem. You may use as many questions as you feel you need to answer the question. However, do not use more questions than you feel you need.

Questions	Answers
1. What is Mr. Smith's salary?	1. His salary is $8000 per year.
2. How much education has he had?	2. He is a high school graduate. He wanted to go to college, but his family did not have enough money to send him.
3. Is Mr. Smith married or single?	3. Mr. Smith is married. He has three children.
4. How old is Mr. Smith?	4. He is 46 years old.
5. What kind of job does he have?	5. He is Supervisor of a loading dock.
6. How long has he had his present job?	6. He has had his job for 11 years.
7. How long has he lived in his present home?	7. He has been in his present home for 6 years.
8. What is the name of the city?	8. The name of the city is Springton.
9. Where in the city does Mr. Smith live?	9. He lives at the North End of the city near the high school.
10. How many banks are there in the city?	10. There are two banks: the First National and the State Bank.
11. Does Mr. Smith have a bank account?	11. No, Mr. Smith does not have a bank account.
12. Where is the Grant Co.?	12. The Grant Co. is just west of the city, off the main highway.
13. Where is the Williams Corp.?	13. The Williams Corp. is north of the city, just accross the river.
14. Where is Gibbons Metals?	14. Gibbons Metals is near the center of the city.

Problem 19
(Continued)

Questions	Answers
15. How long has Grant and Co. been in the city?	15. Grant Co. is the oldest firm in the city. It has been there over 60 years.
16. How long has the Williams Corp. been in the city?	16. The Williams Corp. has been in the city about 15 years.
17. How long has Gibbons Metals been in the city?	17. Gibbons Metals opened its plant in this city 4 years ago.
18. How much education does Grant Co. require of its employees?	18. The policy of Grant Co. in recent years has been to hire only college graduates. As a result, all employees under 40 are college graduates. Some of the employees over 40 are only high school graduates, while some others are college graduates
19. How much education does the Williams Corp. require of its employees?	19. The Williams Corp. hires only college graduates for supervisory positions. Employees below supervisor may be high school graduates only.
20. How much education does the Gibbons Metals require of its employees?	20. Gibbons Metals has no policy on education. Its employees may have any amount of education.

Solution: Mr. Smith works for the Grant Co.

Problem 25

Instructions and Corresponding Questions and Answers

The figure here is composed of overlapping geometric figures and lines, forming twenty-five areas. You will notice that an area is any enclosed part of the figure that does not have a line through it. One of the areas has been picked at random. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from other areas. You may discover these facts by using any of the questions you like.

Proceed by reading over all the questions. Then decide on the first question you would like to have answered, and write its number on the page provided. Then take the card from the folder, and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number and take the card from the folder. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may use as many of the cards as you need to find the correct area, but don't draw any more than you need.

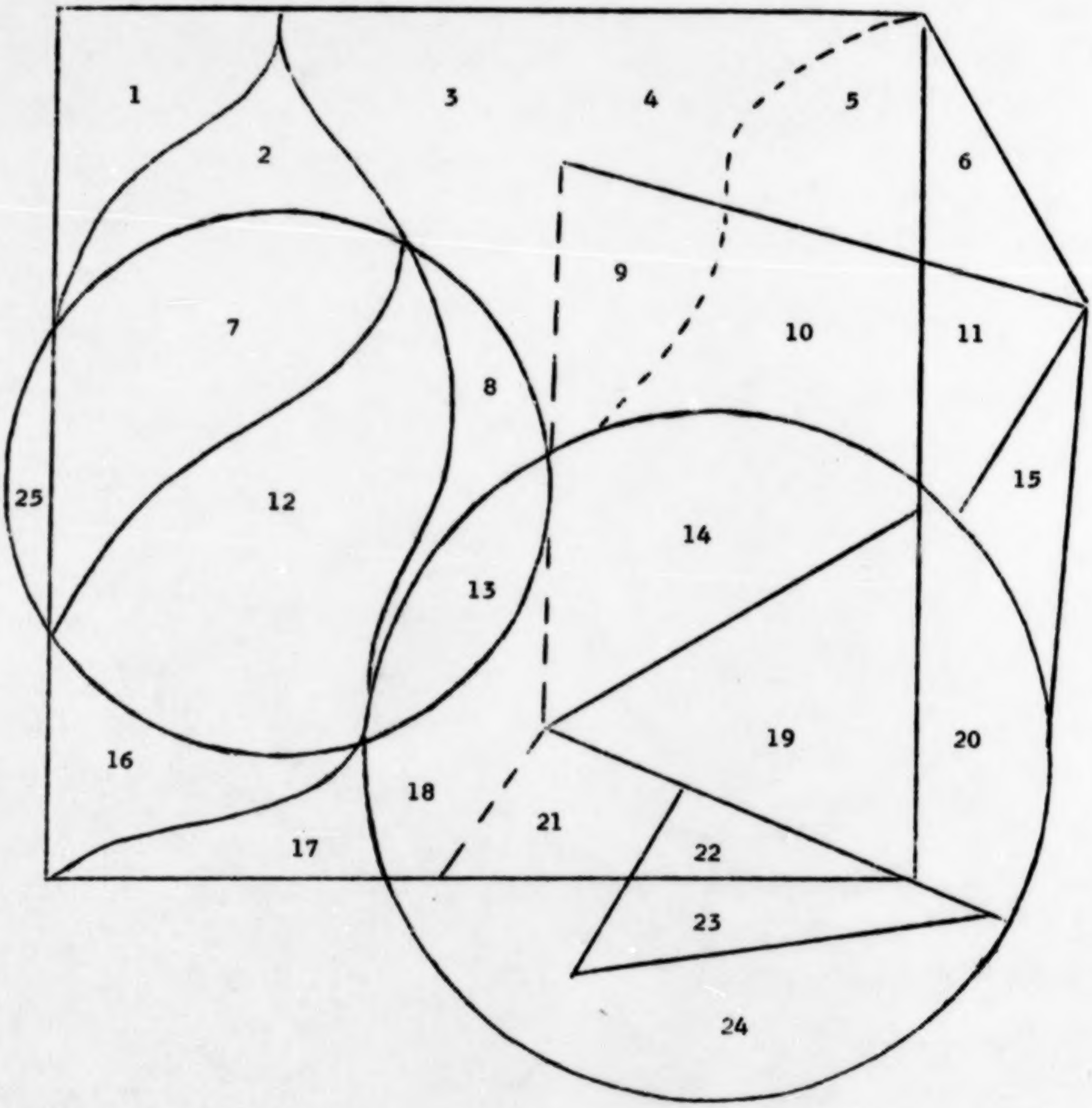
Questions	Answers
1. Is it in the large square?	1. Yes, it is in the large square.
2. Is it in the large circle?	2. No, it is not in the large circle.
3. Is it in the small circle?	3. No, it is not in the small circle.
4. Does the area have at least one right angle?	4. No, it does not have at least one right angle. e
5. Is the area a rectangle?	5. No, the area is not a rectangle.
6. Is the area a triangle?	6. No, the area is not a triangle.
7. Is it outside the large square?	7. No, it is not outside the large square.
8. Is it outside the large circle?	8. Yes, it is outside the large circle.
9. Is it to the left of the straight dotted line?	9. No, it is not to the left of the straight dotted line.
10. Is it to the right of the straight dotted line?	10. Yes, it is to the right of the straight dotted line.
11. Does it have all straight line boundaries?	11. No, it does not have all straight line boundaries.
12. Does it have all curved boundaries?	12. No, it does not have all curved boundaries.
13. Does the area have at least two boundaries?	13. Yes, the area does have at least two boundaries.
14. Does the area have at least three boundaries?	14. Yes, the area does have at least three boundaries.
15. Does the area have at least four boundaries?	15. Yes, the area does have at least four boundaries.

Problem 25
(Continued)

	Questions		Answers
16.	Does the area have at least five boundaries?	16.	No, the area does not have at least five boundaries.
17.	Does it have at least two straight line boundaries?	17.	Yes, it has at least two straight line boundaries.
18.	Does it have at least three straight line boundaries?	18.	No, it does not have at least three straight line boundaries.
19.	Does it have at least four straight line boundaries?	19.	No, it does not have at least four straight line boundaries.
20.	Does it have at least two curved boundaries?	20.	Yes, it does have at least two curved boundaries.
21.	Does it have at least three curved boundaries?	21.	No, it does not have at least three curved boundaries.
22.	Does it have at least four curved boundaries?	22.	No, it does not have at least four curved boundaries.
23.	Does it have a dotted boundary?	23.	Yes, it does have a dotted boundary.
24.	Does it have two dotted boundaries?	24.	No, it does not have two dotted boundaries.
25.	Does it have a straight line boundary?	25.	Yes, it does have a straight line boundary.
26.	Does it have a curved boundary?	26.	Yes, it does have a curved boundary.

Solution: The pre-selected area is #10.

Problem 25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Appendix I

Problem 31 A

Instructions and Corresponding Questions and Answers

At Spencer High School the annual fall dance is about to be held. A dance committee has been selected to make the necessary arrangements. Both boys and girls are on the committee. A part of the committee is to take care of the refreshments for the evening and another part will look after the sale of the tickets for the dance. The list of the girls on the dance committee involved in the sale of tickets has been lost. From the other information available, which you will find in the questions, your object will be to discover the number of girls involved in the sale of tickets.

Questions	Answers
1. Is Spencer High School the only coeducational school in the city?	1. No.
2. How many boys attend Spencer High?	2. 240 boys attend Spencer High.
3. How many boys are on the dance committee?	3. 10.
4. Are there more girls than boys at this school?	4. Yes.
5. How many students on the dance committee are assigned to supplying the refreshments?	5. 14.
6. What is the total number of students on the fall dance committee?	6. 25.
7. How much time would the committee as a whole spend in preparation for the dance?	7. 275 hours.
8. How much time would the average committee member contribute?	8. 11 hours.
9. How many boys on the committee are involved in the sale of tickets?	9. 6 boys.
10. How many girls are on the refreshment part of the dance committee?	10. 10 girls.

Solution: 5 girls.

Problem 31 B

Instructions and Corresponding Questions and Answers

We have a certain number of objects, M, a part of which, for lack of a better name, will be called C's. The C's are composed of B's and G's. No B is a G and vice versa. Some of the C's also are R's and some others are T's. No R is a T and vice versa. How many G's are also T's?

Questions	Answers
1. Are there C's that are not B's and G's?	1. No.
2. How many B's are C's?	2. 30.
3. How many B's are M's?	3. 120.
4. How many C's are R's?	4. 35.
5. Are there more G's than B's among the M's?	5. Yes.
6. What is the value of k times the C's?	6. 550.
7. What is the total number of C's?	7. 50.
8. How many B's that are C's are also T's?	8. 10.
9. How many G's that are C's are also R's?	9. 15.
10. What is the value of k?	10. 11.

Solution: 5 G's.

Problem 31 C

Instructions and Corresponding Questions and Answers

Assume that X, A, D, P, and S, represent properties among F objects. Not-X, not-A, and so on represent lack of these properties. Out of F objects some of them are X's and some not-X's. The not-X's are formed by not-A's and not-D's. A not-A can not be a not-D and vice versa.

Some of the not-X's also are not-P's and some others are not-S's. A not-P can not be a not-S and vice versa.

How many not-D's are also not-S's?

Questions	Answers
1. Are there not-X's that are A's and D's?	1. No.
2. How many not-A's are F's?	2. 100.
3. Are there more not-D's than not-A's among the F's?	3. Yes.
4. How many not-A's are not-X's?	4. 14.
5. What is the total number of not-X's?	5. 40.
6. How many not-X's are not-P's?	6. 24.
7. What is the value of 1 times the not-X's?	7. 440.
8. What is the value of 1?	8. 11.
9. How many not-D's that are not-X's are also not-P's?	9. 20.
10. How many not-A's that are not-X's are also not-S's?	10. 10.

Solution: 6 not-D's.

Problem 31 D

Instructions and Corresponding Questions and Answers

From R objects L have been selected. These objects are formed by A and B objects. No A can also be a B and vice versa. Some of the L objects are also M and some others N. No M can also be an N and vice versa.

How many N's are also B's?

Questions	Answers
1. How many A's are R's?	1. W.
2. What is the total number of L's?	2. E+F+H+I = X+Y = P+Q = L
3. How many L's are M's?	3. E+F = X
4. How many A's are L's?	4. E+H = P
5. Are there more B's than A's among the R's?	5. Yes.
6. Are there L's that are not B's and A's?	6. No.
7. How many B's that are L are also M?	7. F.
8. How many A's that are L are also N?	8. H.
9. What is the value of k?	9. T.
10. What is the value of k times the L's?	10. Z.

Solution: I

Problem 33 A

Instructions and Corresponding Questions and Answers

A drug-store owner receives different types of magazines that have different prices. Some cost less than 25¢, some cost more than 25¢. You are to find out how many mystery magazines cost more than 25¢. Read all the questions carefully.

Questions	Answers
1. What kinds of magazines does the store receive?	1. Mystery, sports and mechanical magazines.
2. Has the store more magazines about mechanics than sports?	2. No.
3. Has the store more magazines about mechanics than mystery magazines?	3. Yes.
4. How many different prices do the magazines have?	4. 3.
5. What is the total number of magazines which the store receives?	5. 60.
6. How many magazines cost 25¢ or more?	6. 52.
7. How many mechanics magazines cost less than 25¢?	7. 5.
8. Do all the sports magazines cost less than 25¢?	8. No
9. How many sports magazines and mechanics magazines cost less than 25¢?	9. 6.
10. Do all the mystery magazines cost less than 25¢?	10. No.
11. How many sports magazines are received?	11. 30.
12. How many mechanics magazines are received?	12. 20.
13. How many mystery magazines are received?	13. 10.

Solution: 8 mystery magazines.

Problem 33 B

Instructions and Corresponding Questions and Answers

A number of objects have been collected into one general classification and are called T's. There are, however, various types of T's. Each of these types are further subdivided into E's and F's. Your task is to determine how many objects of S type are also F's.

Questions	Answers
1. How many types of objects are there?	1. S, A, and P.
2. Are there more A's than S's?	2. Yes.
3. Are there more P's than A's?	3. Yes.
4. How many objects are F's?	4. 33.
5. What is the total number of objects?	5. 45.
6. How many properties can each type have?	6. F and E.
7. How many P's and A's are E's?	7. 8.
8. How many P's are also E's?	8. 3.
9. How many A's are also E's?	9. 5.
10. How many P's are there?	10. 20.
11. How many S's are there?	11. 10.
12. Are all S's also F's?	12. No.
13. How many A's are there?	13. 15.

Solution: 6 objects of S type are also F's.

Problem 33 C

Instructions and Corresponding Questions and Answers

Objects according to their properties have been classified G's and not-G's. The not-G's are composed of different types. Each of these types may be either not-P's or not-Q's. Your task is to discover how many not-A's are also not-Q's.

Questions	Answers
1. How many types of objects are there?	1. Not-A's, not-B's, not-C's.
2. What is the total number of objects?	2. 30.
3. How many properties can each type have?	3. Not-P and not-Q.
4. Are there more not-B's than not-A's?	4. Yes.
5. Are there more not-C's than not-B's?	5. Yes.
6. How many not-B's and not-C's are also not-P's?	6. 4.
7. How many not-C's are also not-P's?	7. 1.
8. How many not-B's are also not-P's?	8. 3.
9. How many objects are not-Q's?	9. 25.
10. How many not-A's are there?	10. 5.
11. How many not-C's are there?	11. 15.
12. How many not-B's are there?	12. 10.
13. Are all not-A's also not-Q's?	13. No.

Solution: 4 not-A's are also not-Q's.

Problem 33 D

Instructions and Corresponding Questions and Answers

A number of objects have been collected into one general classification and are called W's. There are various types of W's. Each of these types is further subdivided into D's and E's. Your task is to determine how many objects of A type are also E's.

Questions	Answers
1. How many types of objects are there?	1. A, B, C.
2. Are there more C's than B's?	2. Yes.
3. Are there more B's than A's?	3. Yes.
4. How many properties can each type have?	4. D and E.
5. What is the total number of objects?	5. R + S + T = M + N + K + L + P + Q.
6. How many objects are E's?	6. N + L + Q = E.
7. How many B's are also E's?	7. L.
8. How many C's are also E's?	8. Q.
9. How many C's and B's are D's?	9. P + K.
10. Are all A's also E's?	10. No.
11. How many B's are there?	11. S = K + L.
12. How many C's are there?	12. T = P + Q.
13. How many A's are there?	13. R = M + N.

Solution: N objects of A type are also E's.

Problem 35 A

Instructions and Corresponding Questions and Answers

A college choral group is composed of freshmen, sophomores and juniors. The chorus has three voices or parts which are high, medium, and low. The questions and answers below give vital information concerning the group. From these facts you are to find the number of juniors singing the middle or medium part.

Questions	Answers
1. How many juniors are in this college?	1. 1567.
2. How many freshmen are in the chorus?	2. 23.
3. How many sophomores are in the middle voice?	3. 10.
4. How many chorus members are there?	4. 76.
5. How many girls are in the chorus?	5. 45.
6. How many sophomores are in the chorus?	6. 28.
7. How many juniors sing the high voice?	7. 7.
8. How many freshmen are in this college?	8. 1848
9. How many freshmen sing the high voice?	9. 8.
10. How many low voice members are there?	10. 28.
11. How many sophomores sing the high part?	11. 9.
12. How many pianos does the chorus have?	12. 3
13. How many freshmen sing the low voice?	13. 9.
14. How many chorus members sing the high voice?	14. 24.
15. How many juniors are in the low voice section?	15. 10.
16. How many freshmen sing the middle voice?	16. 6.
17. How many sophomores sing the low part?	17. 9.

Solution: 8 juniors.

Problem 35 B

Instructions and Corresponding Questions and Answers

T objects are composed of M, N, and P types. Each of these latter three types may or may not also be Q's, R's and S's. From the questions and answers you can discover the various relationships of these objects. Make use of this available information to determine how many T objects are N's and also S's.

	Questions		Answers
1.	How many S's are A's?	1.	350.
2.	How many Q's are there among the T's?	2.	19.
3.	How many G's are there among the T's?	3.	43.
4.	How many R's are also N's?	4.	8.
5.	What is the total number of T objects?	5.	63.
6.	How many P's are there among the T's?	6.	21.
7.	How many R's are there among the T's?	7.	24.
8.	How many Q's are also M's?	8.	5.
9.	How many R's are also M's?	9.	10.
10.	How many S's are also M's?	10.	2.
11.	How many Q's are A's?	11.	400.
12.	How many R's are also P's?	12.	6.
13.	How many Q's are also N's?	13.	3.
14.	How many S's are also P's?	14.	4.
15.	How many M's are among the T's?	15.	17.
16.	How many Q's are also P's?	16.	11.
17.	How many H's among the A's?	17.	2.

Solution: 14 T objects are N's and also S's.

Problem 35 C

Instructions and Corresponding Questions and Answers

A class of objects is distinguished by calling some B's and some others not-B's depending on the possession or non-possession of a certain property. The not-B's are further distinguished into not-X's, not-Y's, and not-Z's. Each of these latter may also be a not-D, not-E, or not-F. From the accompanying questions and answers you can discover the relationships that exist between these objects. Make use of the information available to determine how many not-B objects are not-Y's and also not-F's.

	Questions		Answers
1.	How many not-D's are not-A's?	1.	150.
2.	How many not-F's are also not-X's?	2.	7.
3.	How many not-E's are there among the not-B's?	3.	15.
4.	How many not-G's are there among the not-B's?	4.	30.
5.	What is the total number of not-B's?	5.	45.
6.	How many not-E's are also not-Y's?	6.	6.
7.	How many not-D's are there among the not-B's?	7.	6.
8.	How many not-F's are not-A's?	8.	100.
9.	How many not-E's are also not-Z's?	9.	5.
10.	How many not-D's are also not-Y's?	10.	2.
11.	How many not-F's are also not-Z's?	11.	9.
12.	How many not-X's are there among the not-B's?	12.	12.
13.	How many not-D's are also not-Z's?	13.	3.
14.	How many not-H's are there among the not-A's?	14.	$2 \log \cos 30^\circ$
15.	How many not-E's are also not-X's?	15.	4.
16.	How many not-Z's are there among the not-B's?	16.	17.
17.	How many not-D's are also not-X's?	17.	1.

Solution: 8 not-B objects are not-Y's and also not-F's.

Problem 35 D

Instructions and Corresponding Questions and Answers

A group of L objects taken from a larger group of M objects is composed of objects of the kind A, B and C. If an object is an A, it can not be a B or C. If an object is a B, it can not be an A and/or C. If an object is a C, it can not be a B and/or A. That is, A, B, and C are mutually exclusive. The same L objects also have properties D, E and F which are mutually exclusive.

From the questions below you are to find how many of the B's are also F's.

Questions	Answers
1. How many F's are in J?	1. U.
2. How many L's are D's?	2. M + N + O = X.
3. What is the number of L's?	3. M+N+O+R+Q+P+S+T+V = X+Y+Z = G+H+I = L.
4. How many E's are B's?	4. Q.
5. How many L's are K's?	5. W.
6. How many D's are in M?	6. X - M + O.
7. How many L's are E's?	7. R + Q + P = Y
8. How many F's are A's?	8. S.
9. How many E's are A's?	9. R.
10. How many D's are A's?	10. M.
11. How many L's are C's?	11. O + P + V = I.
12. How many F's are C's?	12. V.
13. How many L's are A's?	13. M + R + S = G.
14. How many D's are C's?	14. O.
15. How many U's are M's?	15. U - J.
16. How many D's are B's?	16. N.
17. How many E's are C's?	17. P.

Solution: T of the B's are also F's.

Problem 37 A

Instructions and Corresponding Questions and Answers

At a branch office of the Kraft Cheese Co. in Peoria, Illinois, a certain number of employees are eligible for promotion. Only a certain limited amount of information is available to the personnel manager concerning these people. Part of this information is the number of employees who have attained various levels of education. These levels of education are distributed among various jobs. As the assistant to the personnel manager you are to determine how many administrative employees are eligible for promotion.

Questions	Answers
1. How many employees are eligible for promotion?	1. 20.
2. How many levels of education are represented among the employees eligible for promotion?	2. 1) elementary education 2) high school education 3) college education
3. How many persons does the Kraft branch office employ?	3. 83.
4. How many employees eligible for promotion have attained only a high school education?	4. 7.
5. What is the kind of work which those who have received only an elementary education do?	5. Some are employed on the loading docks while the others do clerical work.
6. How many employees eligible for promotion have received a college education?	6. 5.
7. What kind of work do the employees having only a high school education do?	7. Administrative work.
8. How many employees eligible for promotion have received only an elementary education?	8. 8.
9. Have all the people in administrative work eligible for promotion received only a high school education?	9. No.
10. What kind of work do the people who have a college education do?	10. The do administrative work and are on the board of directors.
11. Are all the eligible employees with only a high school education doing administrative work?	11. Yes.
12. How many eligible employees with only an elementary education work on the loading docks?	12. 3.

Problem 37 A
(Continued)

Questions	Answers
13. How many eligible employees with a college education are on the board of directors?	13. 3.
14. How many eligible employees with only an elementary education do clerical work?	14. 5.

Solution: 9 administrative employees

Problem 37 B

Instructions and Corresponding Questions and Answers

We are interested in a group of objects which will be denoted by the Greek letter Σ. This group is composed of different varieties each of which will be denoted by a capital letter. These varieties may also be subdivided and these last subdivisions will be identified by small letters. Your task, then, is to discover how many objects from the original group (called Σ) are also d's.

Questions	Answers
1. The group Σ was selected from how many objects?	1. 65.
2. How many objects belong to the group denoted by Σ?	2. 12.
3. How many varieties of Σ are there?	3. Three. 1) A, 2) B, 3) C.
4. How many objects of C variety are there?	4. 3.
5. How many objects of A variety are there?	5. 4.
6. How is B subdivided?	6. Subdivided into "g" and "f" types.
7. Are all the objects that are "d" also of A variety?	7. No.
8. How is A subdivided?	8. It is divided in objects of "d" type.
9. How many objects belong to B variety?	9. 5.
10. How many objects of B variety are "f's"?	10. 3.
11. How many objects of B variety are "g's"?	11. 2.
12. How is C subdivided?	12. into "d" and "e" types.
13. How many objects of C variety are "e's"?	13. 2.
14. Are all the objects of A variety "d's"?	14. Yes.

Solution: 5 objects from the original group (called Σ) are also d's.

Instructions for Problem 32 A

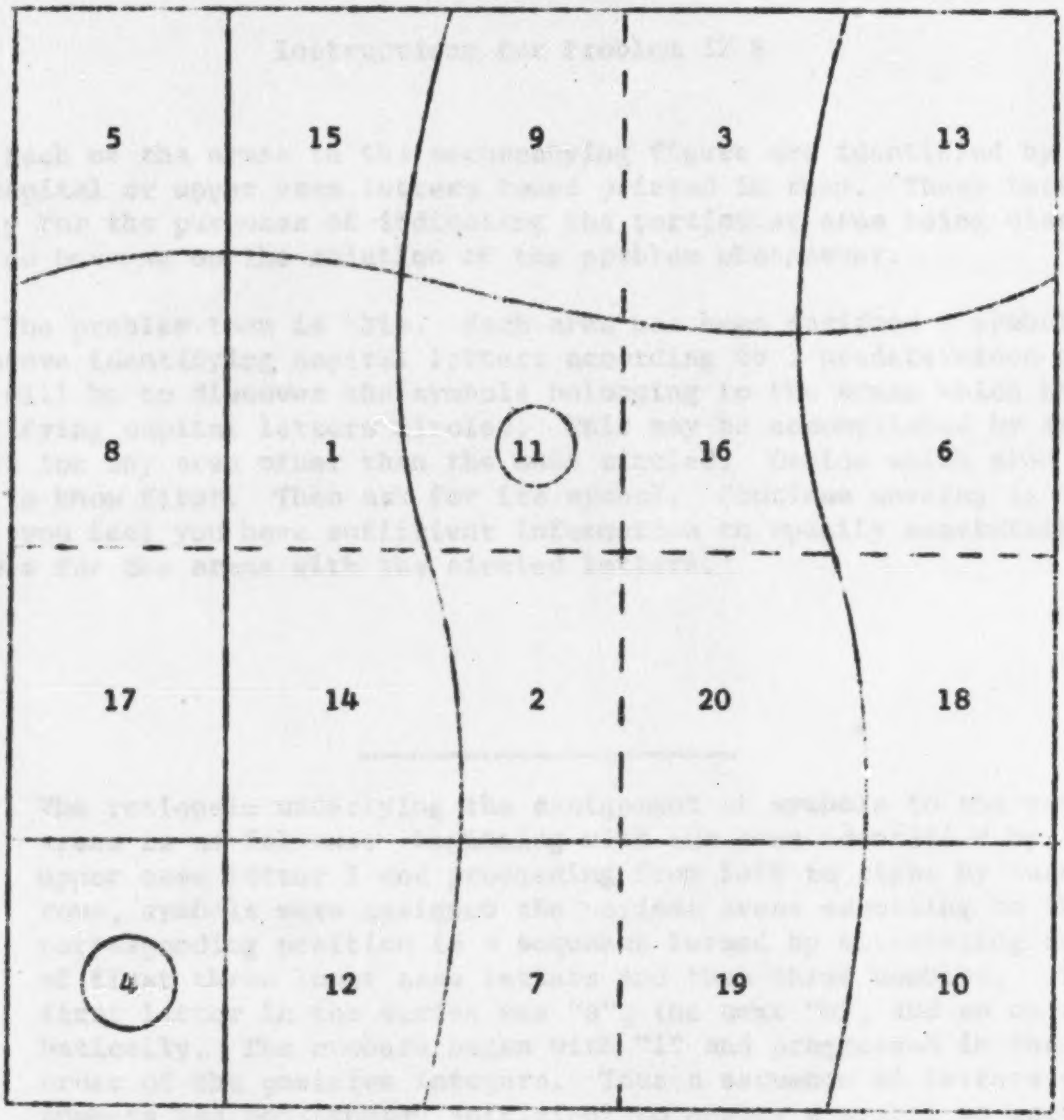
Each of the areas in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a letter according to a predetermined plan. Your task will be to discover the letters belonging to the areas which have their identifying numbers circled. This may be accomplished by asking the letter for any area other than the ones circled. Decide which area you would like to know first. Then ask for its letter. Continue working in this fashion until you feel you have sufficient information to specify accurately the letters for the areas with the circled numbers.

Note: The rationale underlying the assignment of letters to the various areas is as follows: beginning with the area identified by the number "5" and proceeding from left to right by successive rows, each area was assigned its corresponding letter in the series x, y, z, a, b, c in that order of occurrence, repeating the series as often as is necessary to assign a value to each area.

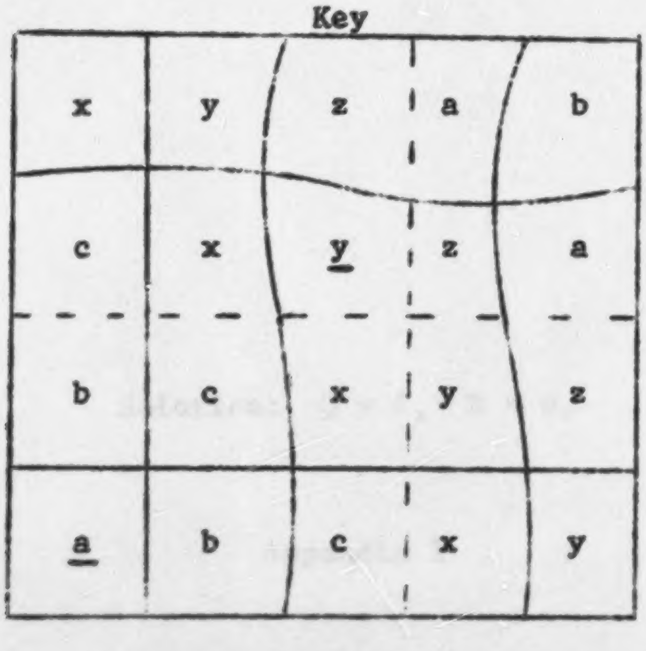
Solution: 11 = y, 4 = a.

Problem 32A



5	15	9	3	13
8	1	⑪	16	6
17	14	2	20	18
④	12	7	19	10

Key



x	y	z	a	b
c	x	y	z	a
b	c	x	y	z
a	b	c	x	y

Appendix I

Instructions for Problem 32 B

Each of the areas in the accompanying figure are identified by means of the capital or upper case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

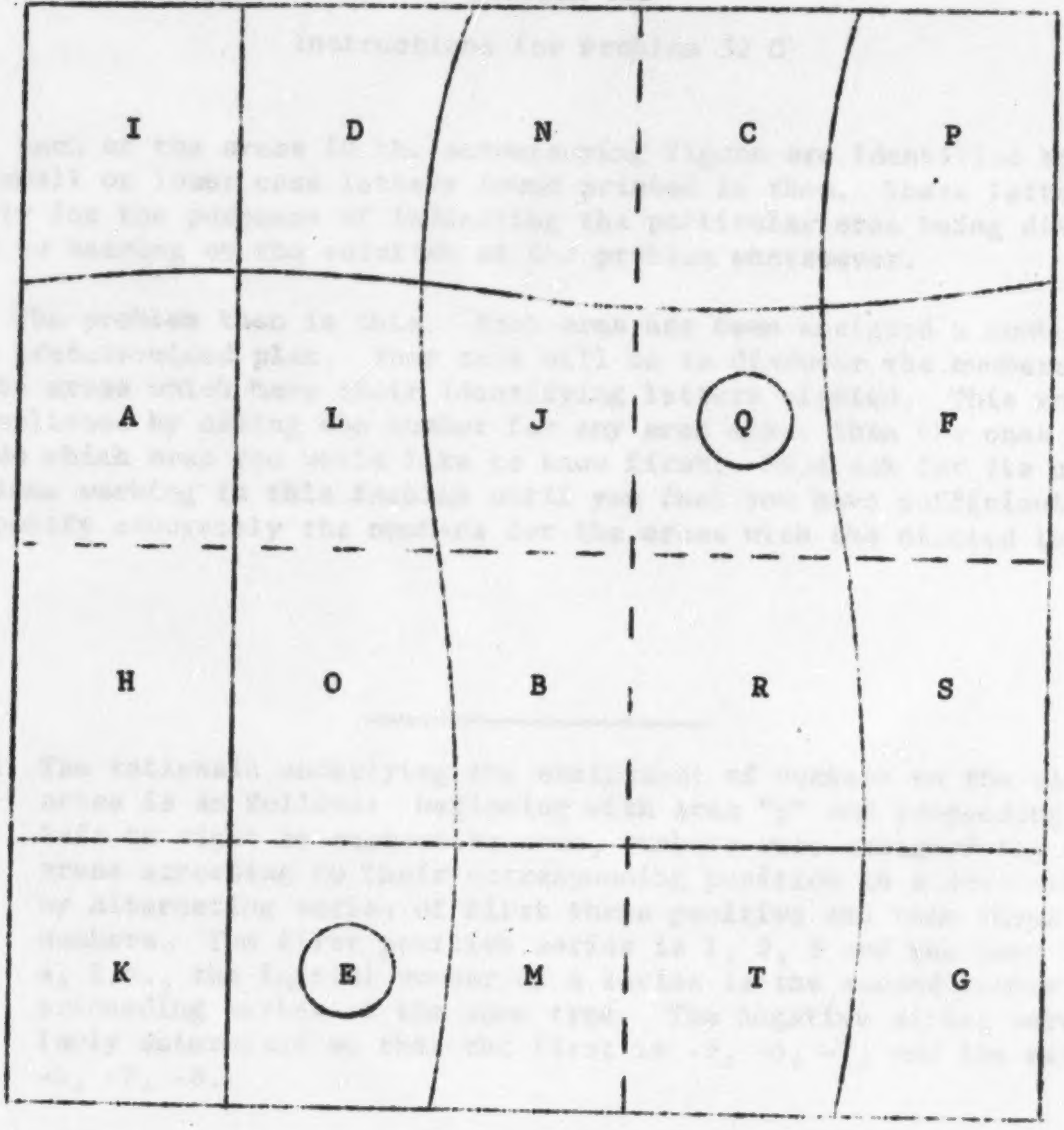
The problem then is this. Each area has been assigned a symbol other than the above identifying capital letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying capital letters circled. This may be accomplished by asking the symbol for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

Note: The rationale underlying the assignment of symbols to the various areas is as follows: beginning with the area identified by the upper case letter I and proceeding from left to right by successive rows, symbols were assigned the various areas according to their corresponding position in a sequence formed by alternating series of first three lower case letters and then three numbers. The first letter in the series was "a", the next "b", and so on alphabetically. The numbers began with "1" and progressed in the normal order of the positive integers. Thus a sequence of letters and numbers was constructed sufficient to assign a symbol to each area.

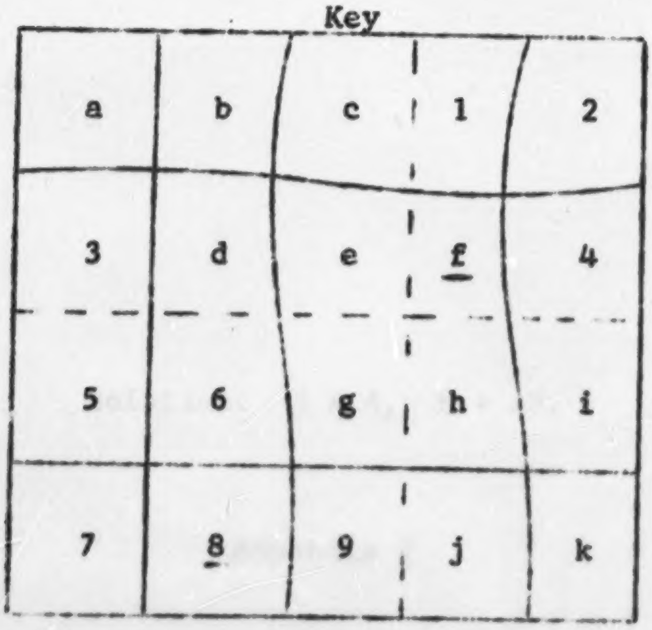
Solution: Q = f, E = 8.

Appendix I

Problem 32B


I
D
N
C
P
A
L
J
Q
F
H
O
B
R
S
K
E
M
T
G

Key


a
b
c
1
2
3
d
e
f
4
5
6
g
h
i
7
8
9
j
k

Instructions for Problem 32 C

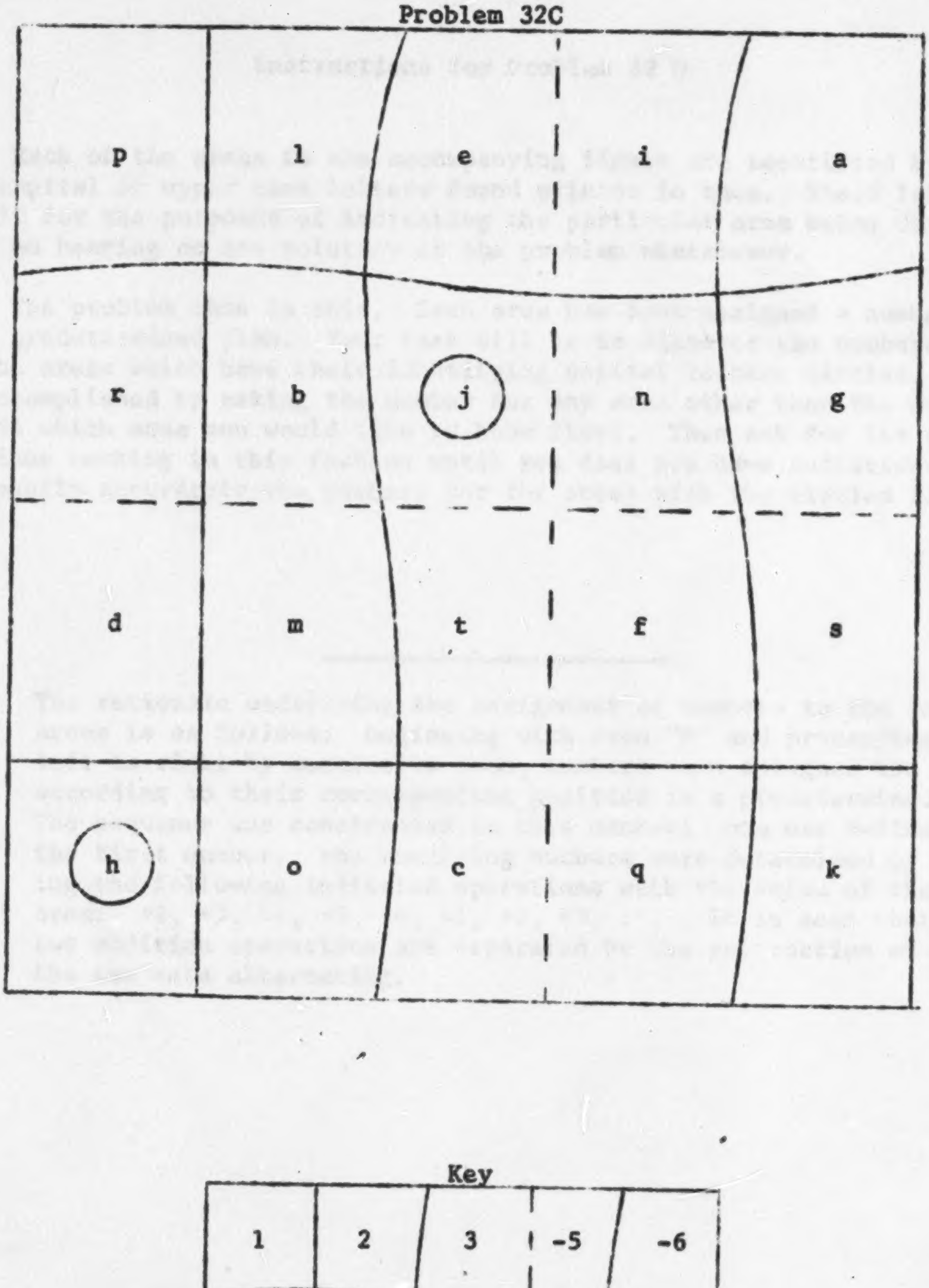
Each of the areas in the accompanying figure are identified by means of the small or lower case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a number according to a predetermined plan. Your task will be to discover the numbers belonging to the areas which have their identifying letters circled. This may be accomplished by asking the number for any area other than the ones circled. Decide which area you would like to know first. Then ask for its number. Continue working in this fashion until you feel you have sufficient information to specify accurately the numbers for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: beginning with area "p" and proceeding from left to right by successive rows, numbers were assigned the various areas according to their corresponding position in a sequence formed by alternating series of first three positive and then three negative numbers. The first positive series is 1, 2, 3 and the next is 2, 3, 4, i.e., the initial number of a series is the second number of the preceeding series of the same type. The negative series were similarly determined so that the first is -5, -6, -7, and the second is -6, -7, -8.

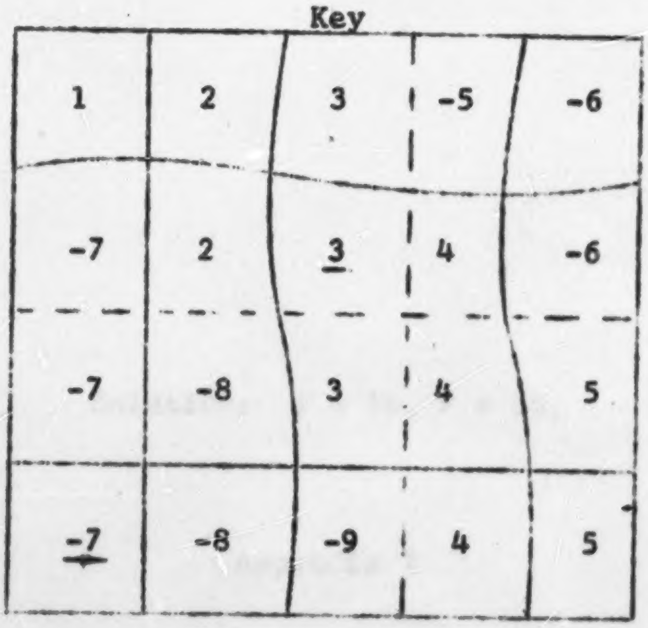
Solution: $j = 3$, $h = -7$.

Problem 32C



p	l	e	i	a
r	b	ⓙ	n	g
d	m	t	f	s
ⓗ	o	c	q	k

Key



1	2	3	-5	-6
-7	2	3	4	-6
-7	-8	3	4	5
-7	-8	-9	4	5

Instructions for Problem 32 D

Each of the areas in the accompanying figure are identified by means of the capital or upper case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

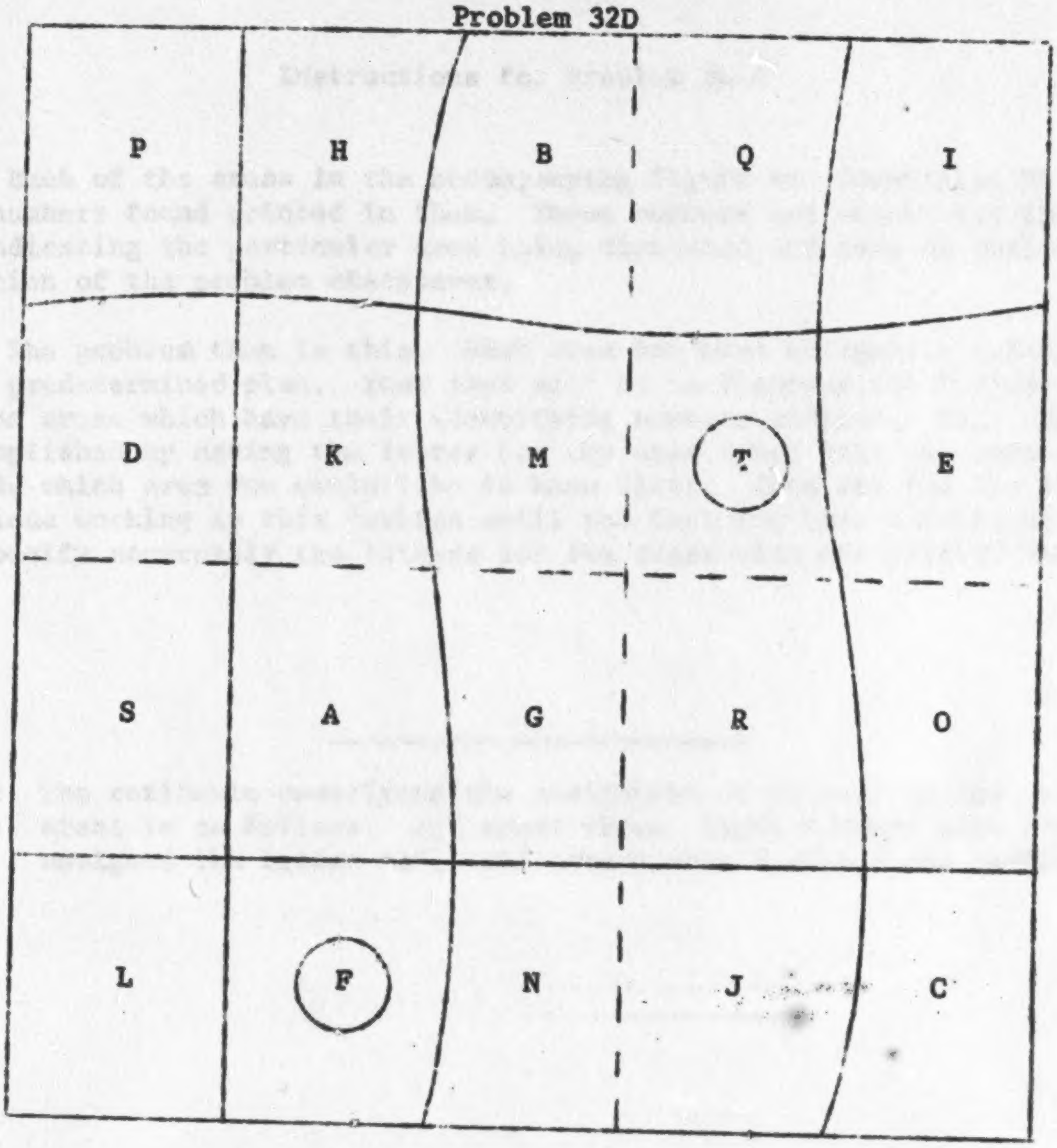
The problem then is this. Each area has been assigned a number according to a predetermined plan. Your task will be to discover the numbers belonging to the areas which have their identifying capital letters circled. This may be accomplished by asking the number for any area other than the ones circled. Decide which area you would like to know first. Then ask for its number. Continue working in this fashion until you feel you have sufficient information to specify accurately the numbers for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: beginning with area "P" and proceeding from left to right by successive rows, numbers were assigned the areas according to their corresponding position in a predetermined sequence. The sequence was constructed in this manner: one was defined to be the first number; the remaining numbers were determined by performing the following indicated operations with the value of the previous area: +2, +3, -1, +3, +4, -1, +2, +3, . . . It is seen that every two addition operations are separated by the subtraction of one with the two sets alternating.

Solution: T = 16, F = 28.

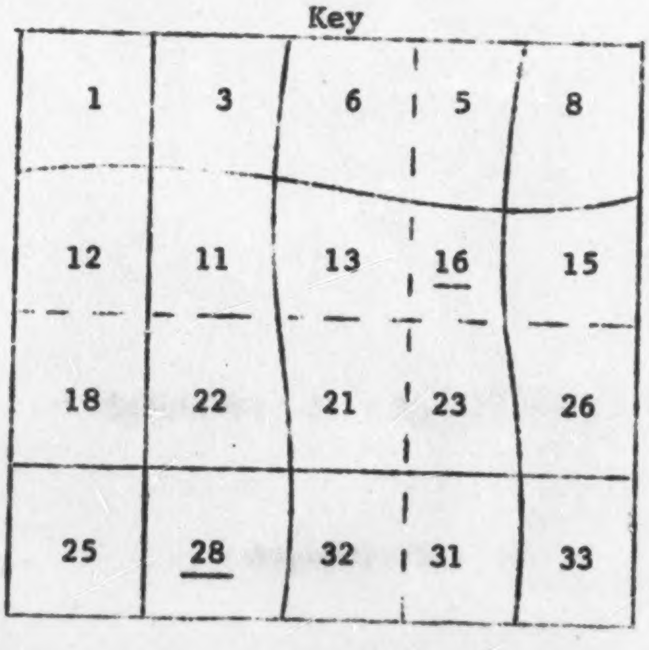
Appendix I

Problem 32D



P	H	B	Q	I
D	K	M	T	E
S	A	G	R	O
L	F	N	J	C

Key



1	3	6	5	8
12	11	13	16	15
18	22	21	23	26
25	28	32	31	33

Appendix I

Instructions for Problem 34 A

Each of the areas in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

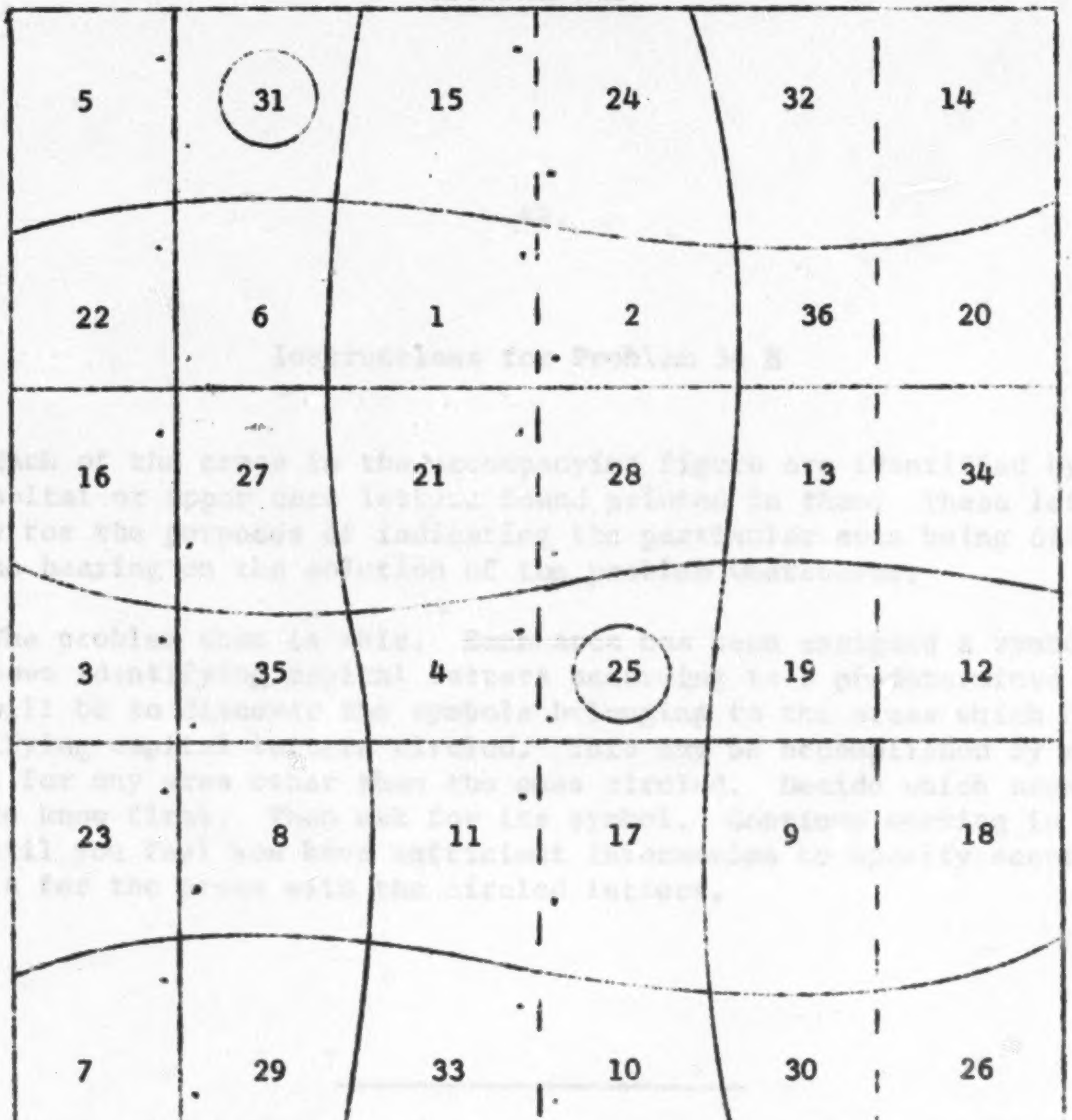
The problem then is this. Each area has been assigned a letter according to a predetermined plan. Your task will be to discover the letters belonging to the areas which have their identifying numbers circled. This may be accomplished by asking the letter for any area other than the ones circled. Decide which area you would like to know first. Then ask for its letter. Continue working in this fashion until you feel you have sufficient information to specify accurately the letters for the areas with the circled numbers.

Note: The rationale underlying the assignment of letters to the various areas is as follows: all areas whose right borders were red were assigned the letter "a", all others were assigned the letter "b".

Solution: 31 = b, 25 = b.

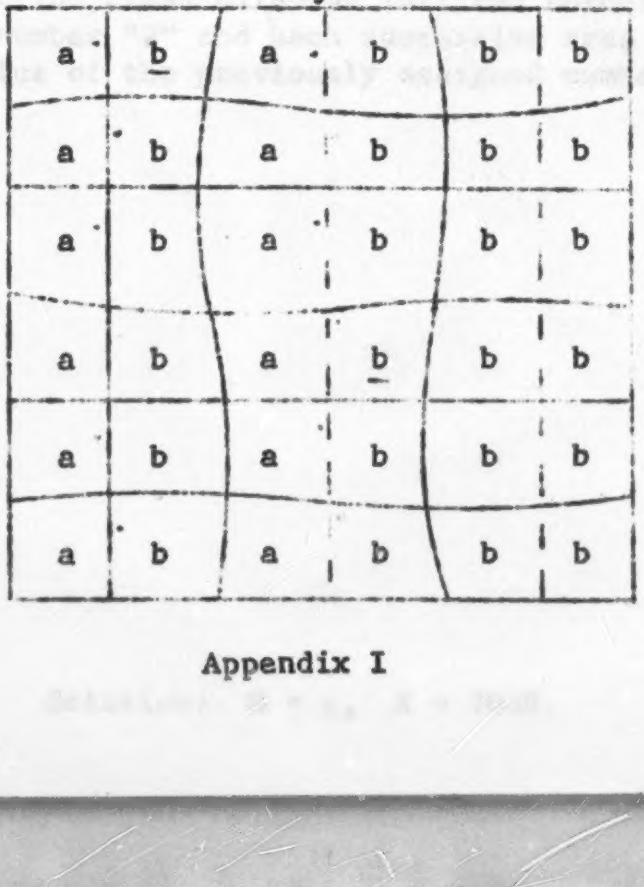
Appendix I

Problem 34A



5	31	15	24	32	14
22	6	1	2	36	20
16	27	21	28	13	34
3	35	4	25	19	12
23	8	11	17	9	18
7	29	33	10	30	26

Key



a	b	a	b	b	b
a	b	a	b	b	b
a	b	a	b	b	b
a	b	a	b	b	b
a	b	a	b	b	b
a	b	a	b	b	b

Note: lines with dots along both sides appeared in red in the original.

Appendix I

Instructions for Problem 34 B

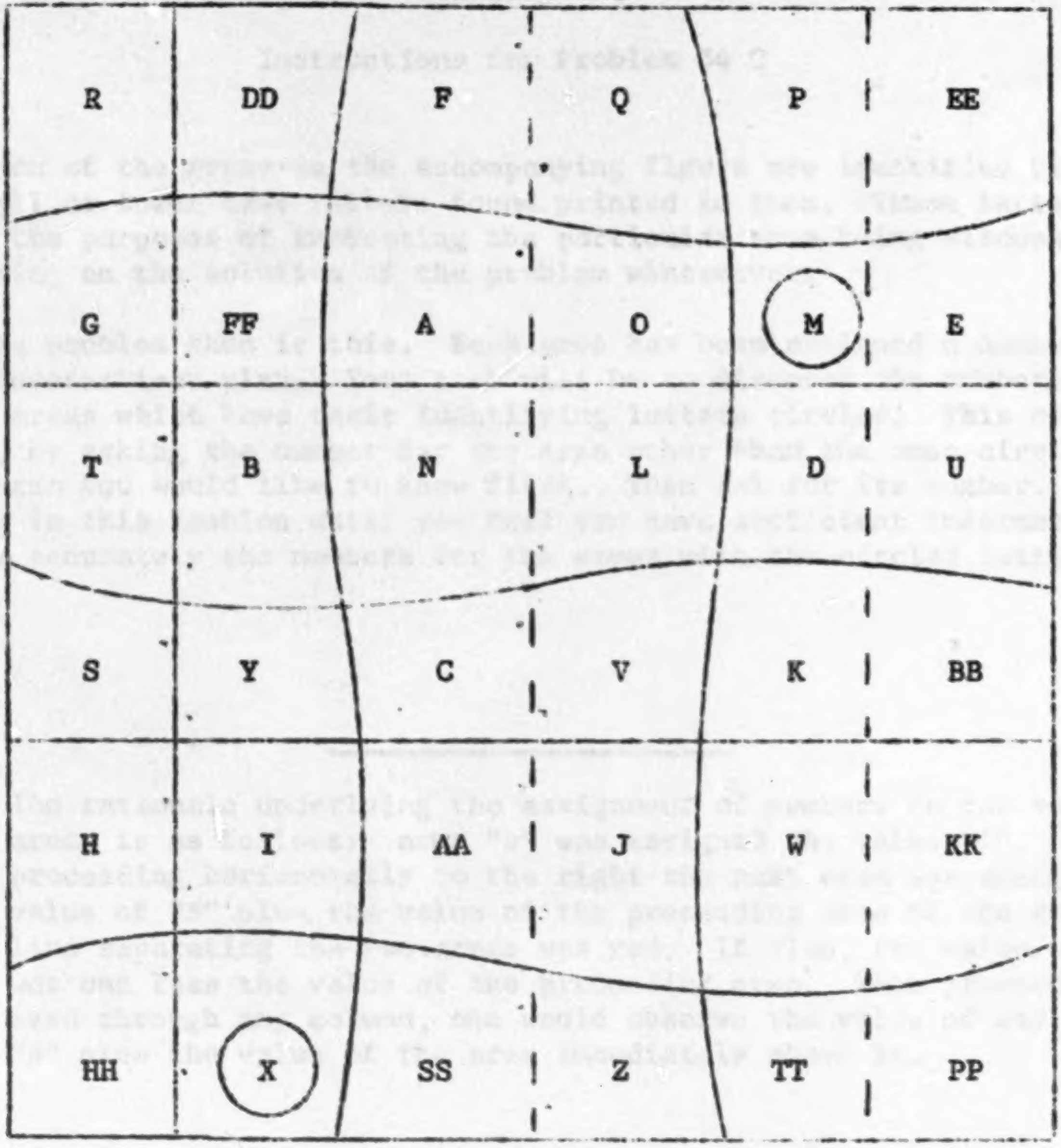
Each of the areas in the accompanying figure are identified by means of the capital or upper case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a symbol other than the above identifying capital letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying capital letters circled. This may be accomplished by asking the symbol for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

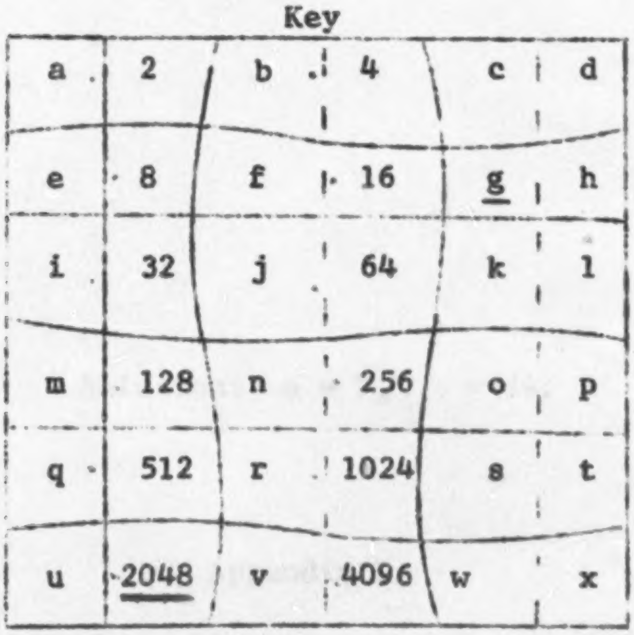
Note: The rationale underlying the assignment of symbols to the various areas is as follows: those areas whose left borders were red received numerical symbols and all others received lower case letter symbols. Beginning with area "R" and proceeding from left to right by successive rows, lower-case letters were assigned the pertinent areas alphabetically from "a" through "x". According to the same procedure, numbers were assigned the areas with the left red border, with area "DD" receiving the number "2" and each successive area receiving a value of twice the value of the previously assigned number.

Solution: M = g, X = 2048.

Problem 34B


R
DD
F
Q
P
EE
G
FF
A
O
M
E
T
B
N
L
D
U
S
Y
C
V
K
BB
H
I
AA
J
W
KK
HH
X
SS
Z
TT
PP

Key


a
2
b
4
c
d
e
8
f
16
g
h
i
32
j
64
k
l
m
128
n
256
o
p
q
512
r
1024
s
t
u
2048
v
4096
w
x

Note: lines with dots appearing along both sides were red in the original.

Instructions for Problem 34 C

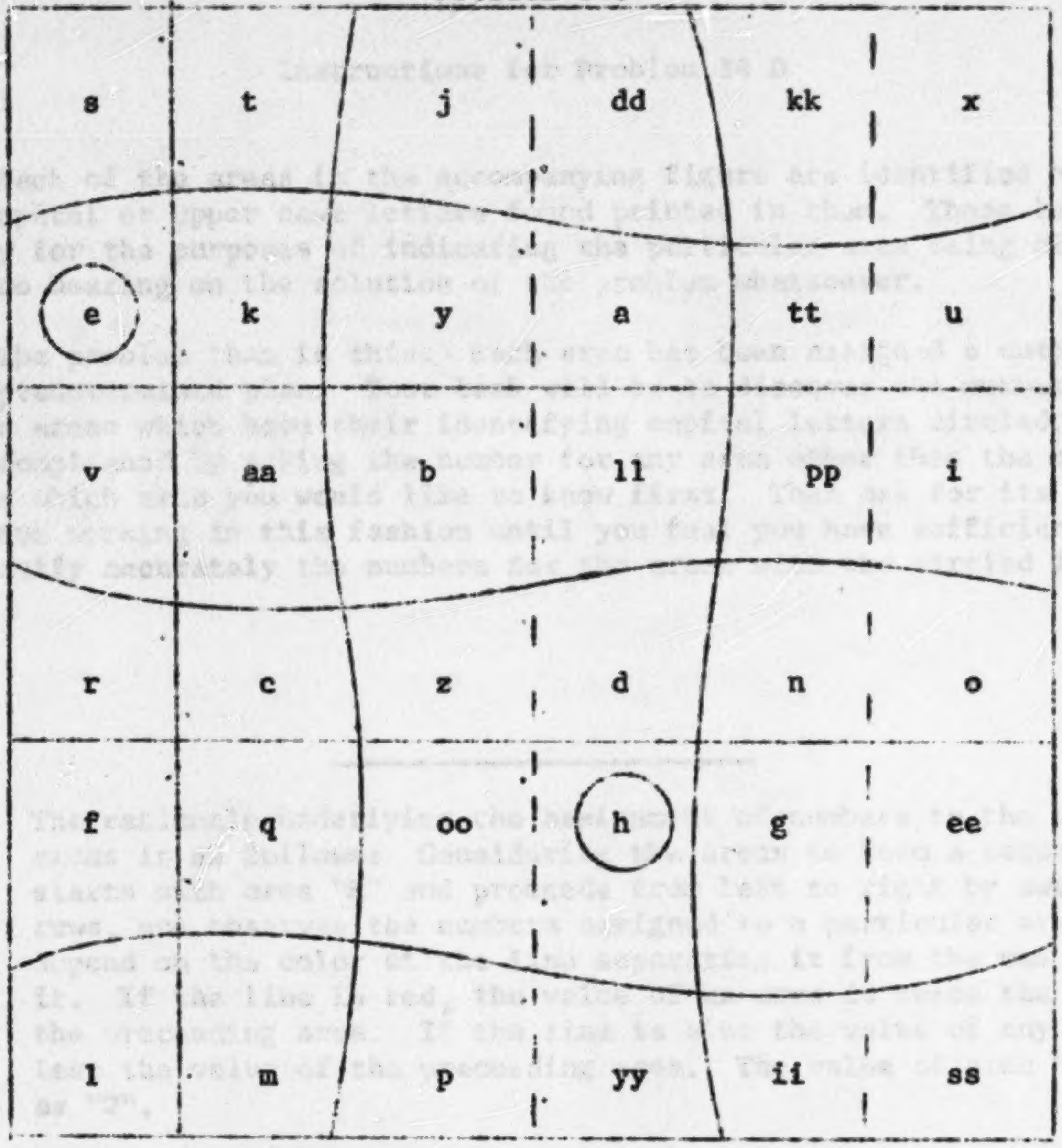
Each of the areas in the accompanying figure are identified by means of the small or lower case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a number according to a predetermined plan. Your task will be to discover the numbers belonging to the areas which have their identifying letters circled. This may be accomplished by asking the number for any area other than the ones circled. Decide which area you would like to know first. Then ask for its number. Continue working in this fashion until you feel you have sufficient information to specify accurately the numbers for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: area "s" was assigned the value "1". Then by proceeding horizontally to the right the next area was assigned a value of "5" plus the value of the preceeding area if the color of the line separating the two areas was red; if blue, the value of any area was one less tha value of the preceeding area. When proceeding downward through any column, one would observe the value of any area to be "6" plus the value of the area immediately above it.

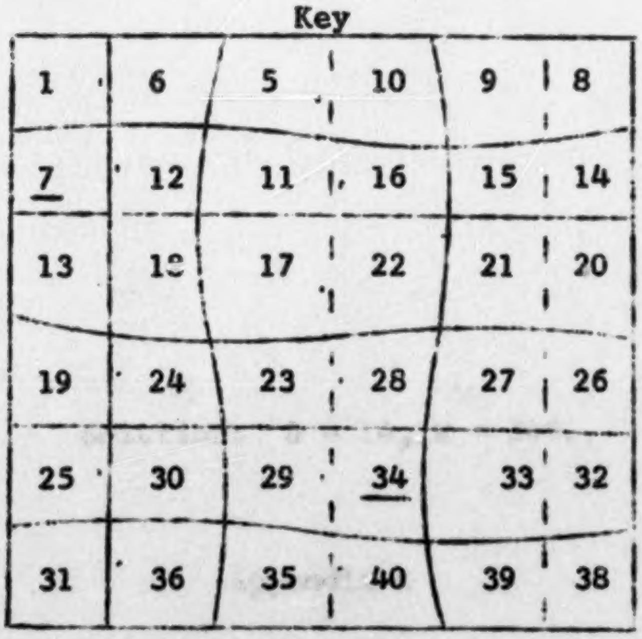
Solution: e = 7, k = 34.

Problem 34C



s	t	j	dd	kk	x
e	k	y	a	tt	u
v	aa	b	ll	pp	i
r	c	z	d	n	o
f	q	oo	h	g	ee
l	m	p	yy	ii	ss

Key



1	6	5	10	9	8
7	12	11	16	15	14
13	18	17	22	21	20
19	24	23	28	27	26
25	30	29	34	33	32
31	36	35	40	39	38

Note: lines with dots along both sides appeared in red in the original.

Appendix I

Instructions for Problem 34 D

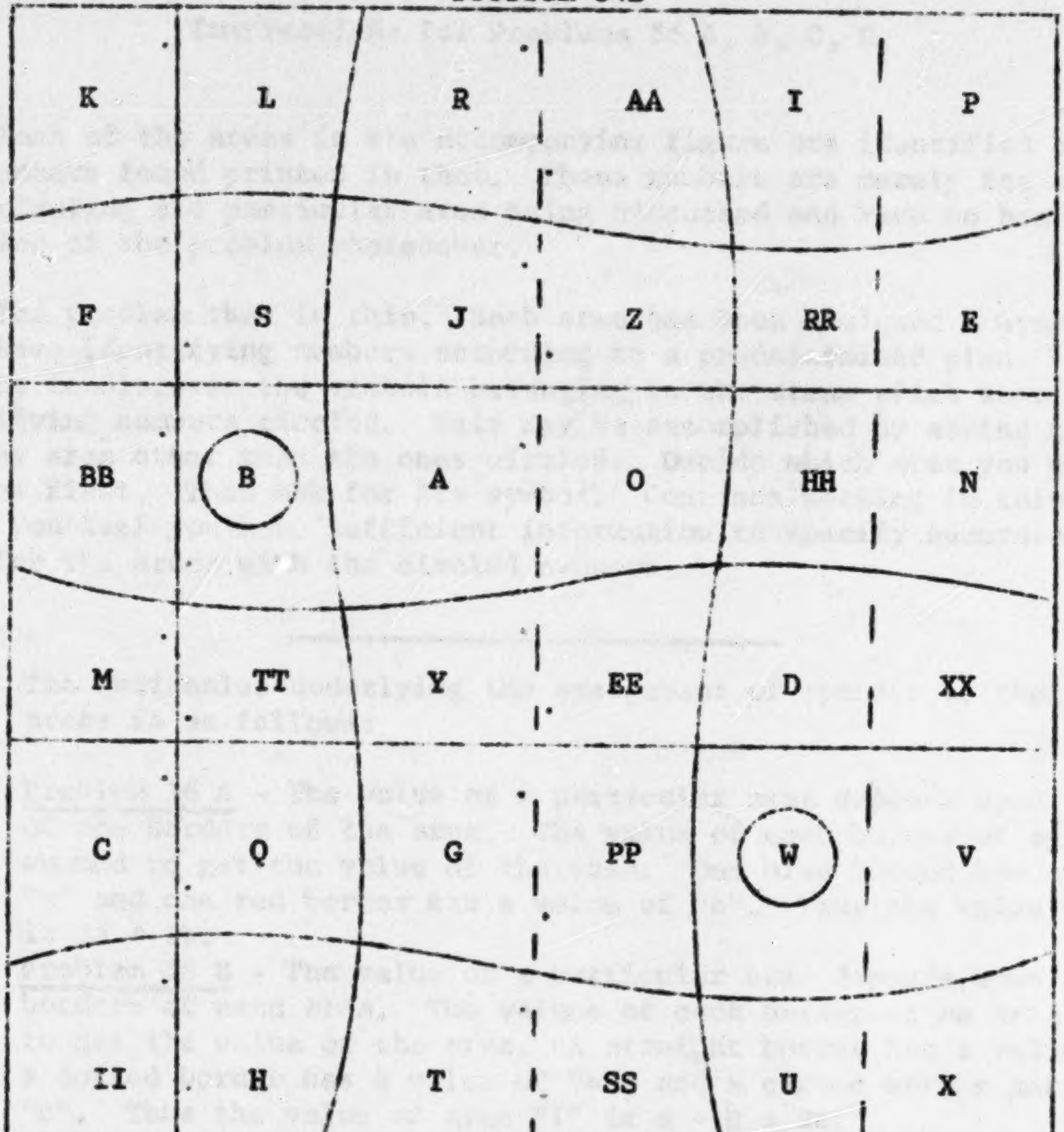
Each of the areas in the accompanying figure are identified by means of the capital or upper case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a number according to a predetermined plan. Your task will be to discover the numbers belonging to the areas which have their identifying capital letters circled. This may be accomplished by asking the number for any area other than the ones circled. Decide which area you would like to know first. Then ask for its number. Continue working in this fashion until you feel you have sufficient information to specify accurately the numbers for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: Considering the areas to form a sequence that starts with area "K" and proceeds from left to right by successive rows, one observes the numbers assigned to a particular area to depend on the color of the line separating it from the one preceeding it. If the line is red, the value of an area is twice the value of the preceeding area. If the line is blue the value of any area is one less the value of the preceeding area. The value of area "K" is given as "2".

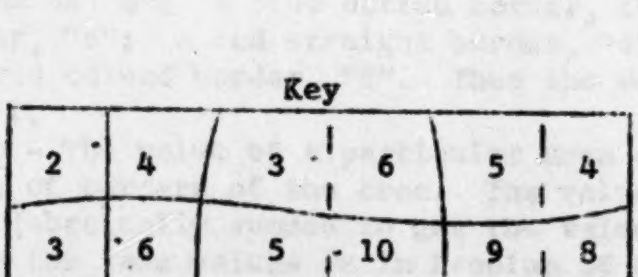
Solution: B = 14, W = 345.

Problem 34D



K	L	R	AA	I	P
F	S	J	Z	RR	E
BB	B	A	O	HH	N
M	TT	Y	EE	D	XX
C	Q	G	PP	W	V
II	H	T	SS	U	X

Key



2	4	3	6	5	4
3	6	5	10	9	8
7	14	13	26	25	24
23	46	45	90	89	88
87	174	173	346	345	344
343	686	685	1370	1369	1368

Note: lines with dots along both sides appeared in red in the original.

Appendix I

Instructions for Problems 36 A, B, C, D.

Each of the areas in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a symbol other than the above identifying numbers according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying numbers circled. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled numbers.

Note: The rationales underlying the assignment of symbols to the various areas is as follows:

Problem 36 A - The value of a particular area depends upon the color of the borders of the area. The value of each border of an area were summed to get the value of the area. One blue border has a value of "a" and one red border has a value of "b". Thus the value of area "2" is 2a + 2b.

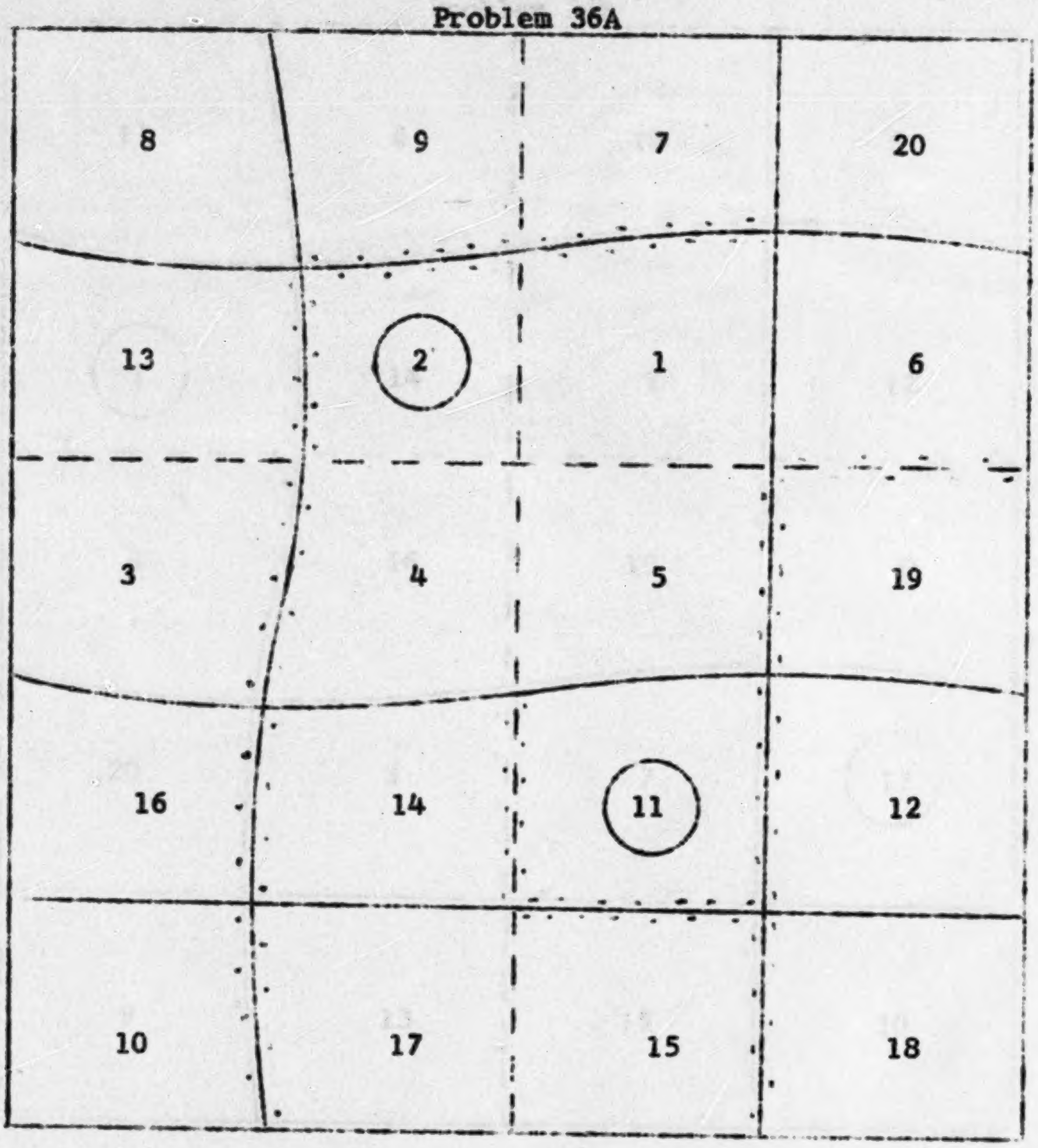
Problem 36 B - The value of a particular area depends upon the type of borders of each area. The values of each border of an area were summed to get the value of the area. A straight border has a value of "a", a dotted border has a value of "b", and a curved border has a value of "c". Thus the value of area "1" is a + B + 2c.

Problem 36 C - The value of a particular area depends upon the color and the type of borders of the area. The values of each border of an area were summed to get the value of the area. A blue straight border has the value of "a"; a blue dotted border, the value of "b"; a blue curved border, "c"; a red straight border, "d"; a red dotted border, "e"; and a red curved border, "f". Thus the value of area "17" is a + b + c + f.

Problem 36 D - The value of a particular area depends upon the color and the type of borders of the area. The values of each border of an area were algebraically summed to get the value of the area. Blue borders have the same values as in Problem 36 C. A straight red border has the value of "-a"; a dotted red border, "-b"; and a curved red border, "-c". Thus the value of area "19" is a + c - b - c.

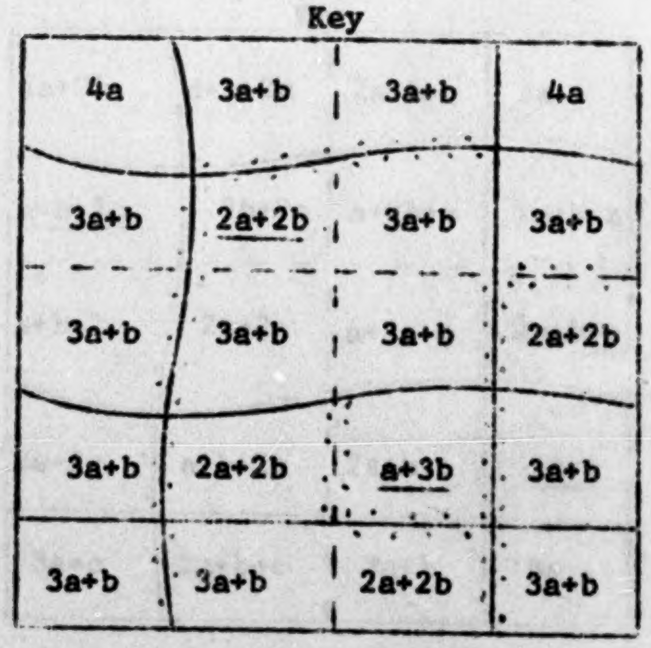
Solutions: Problem 36 A - 2 = 2a + 2b, 11 = a + 3b.
Problem 36 B - 1 = a + b + 2c, 17 = 3a + c.
Problem 36 C - 6 = a + b + c + f, 2 = c + 2d + e.
Problem 36 D - 15 = a + b + c - c, 8 = c - 2a - b.

Problem 36A



8	9	7	20
13	②	1	6
3	4	5	19
16	14	⑪	12
10	17	15	18

Key

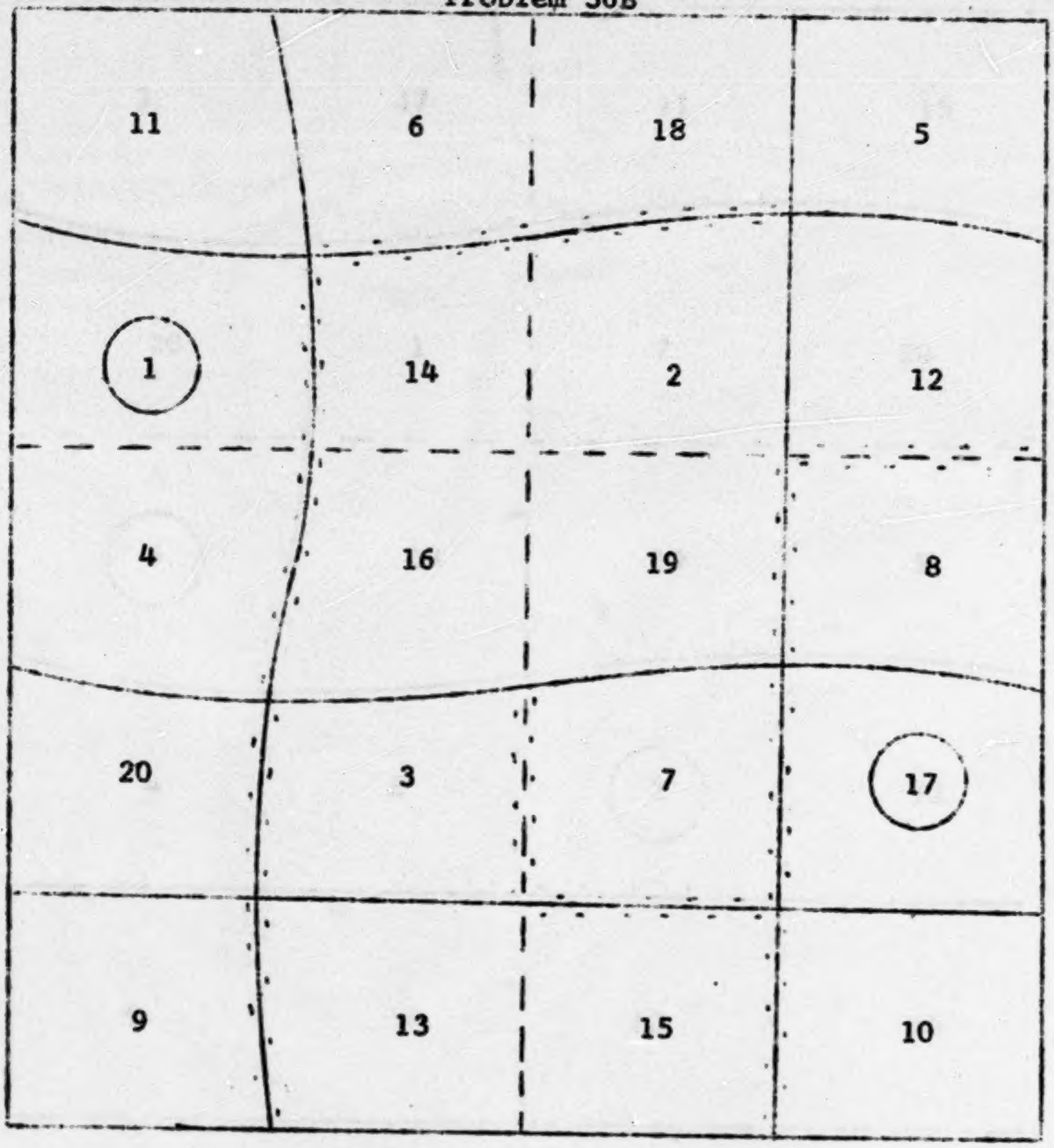


4a	3a+b	3a+b	4a
3a+b	$\underline{2a+2b}$	3a+b	3a+b
3a+b	3a+b	3a+b	2a+2b
3a+b	2a+2b	$\underline{a+3b}$	3a+b
3a+b	3a+b	2a+2b	3a+b

Note: lines or portions of lines with dots along both sides appeared in red in the original.

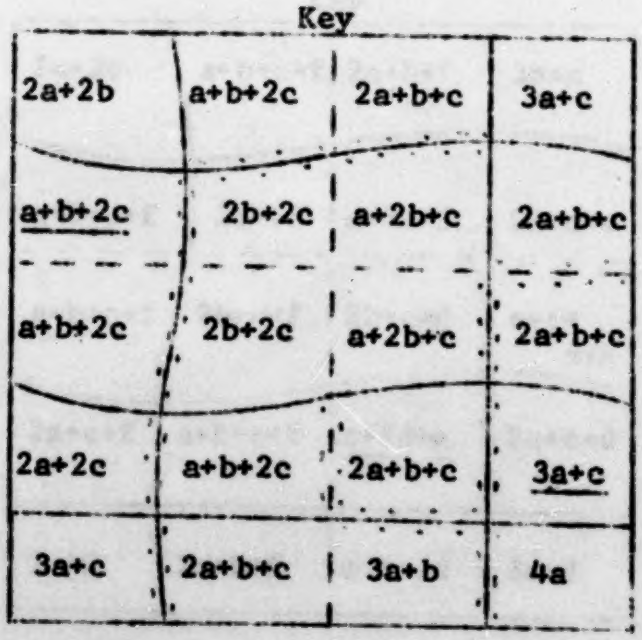
Appendix I

Problem 36B



11	6	18	5
①	14	2	12
4	16	19	8
20	3	7	⑰
9	13	15	10

Key

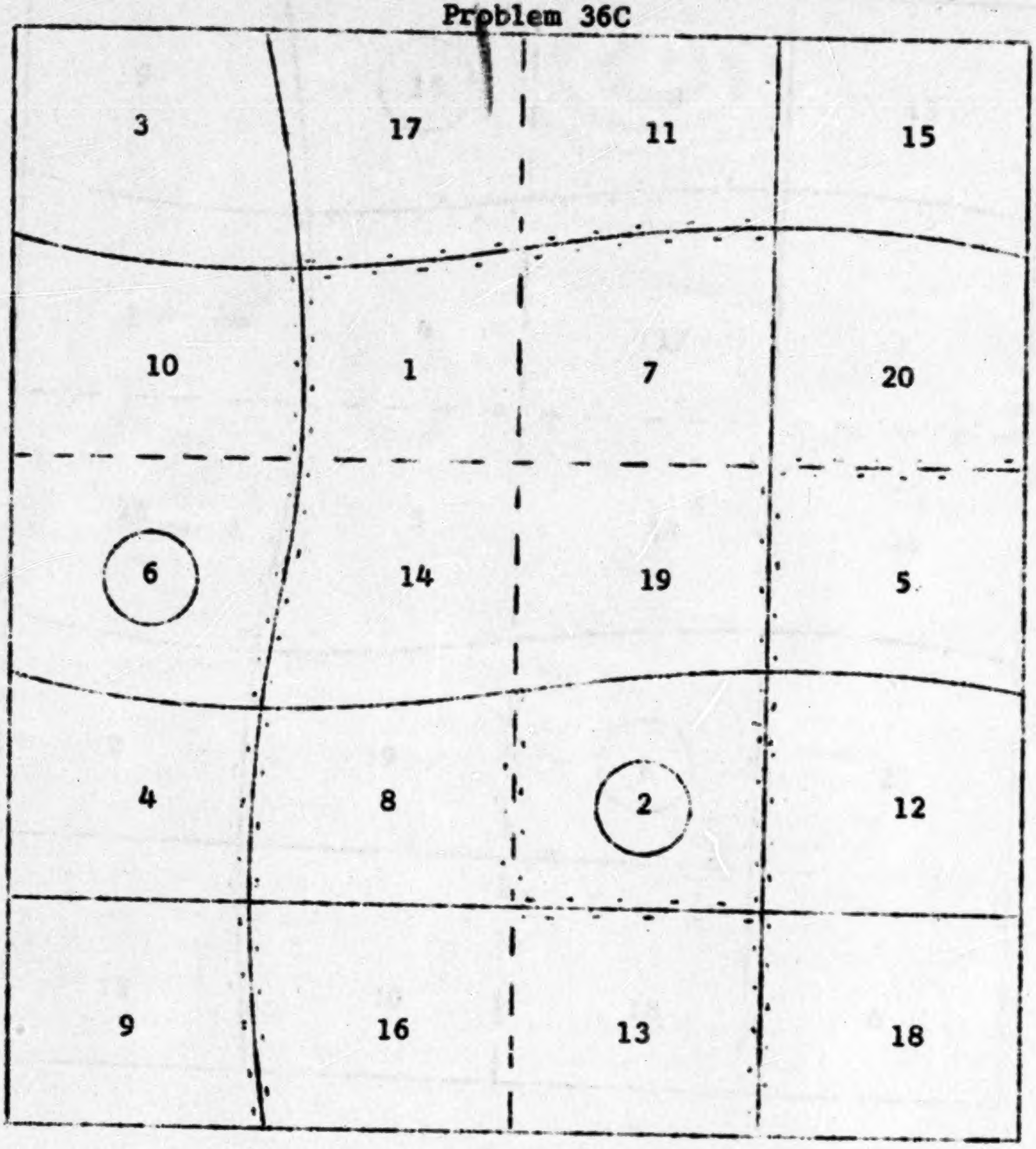


2a+2b	a+b+2c	2a+b+c	3a+c
a+b+2c	2b+2c	a+2b+c	2a+b+c
a+b+2c	2b+2c	a+2b+c	2a+b+c
2a+2c	a+b+2c	2a+b+c	3a+c
3a+c	2a+b+c	3a+b	4a

Note: lines and portions of lines with dots along both sides appeared in red in the original.

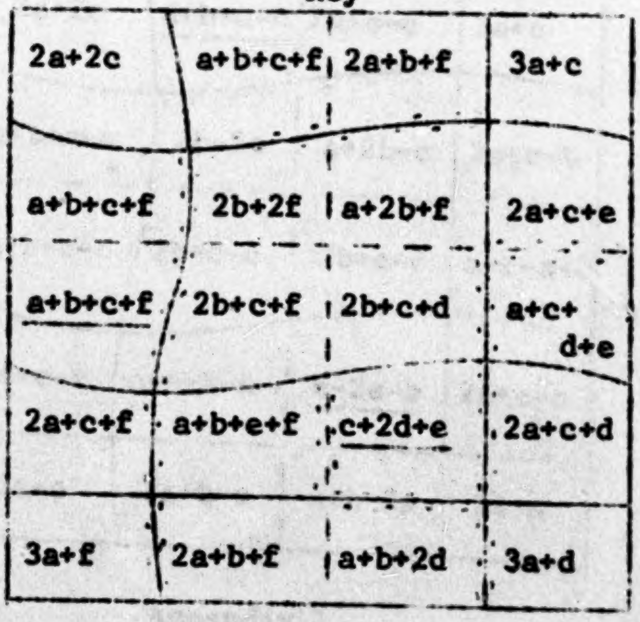
Appendix I

Problem 36C



3	17	11	15
10	1	7	20
6	14	19	5
4	8	2	12
9	16	13	18

Key

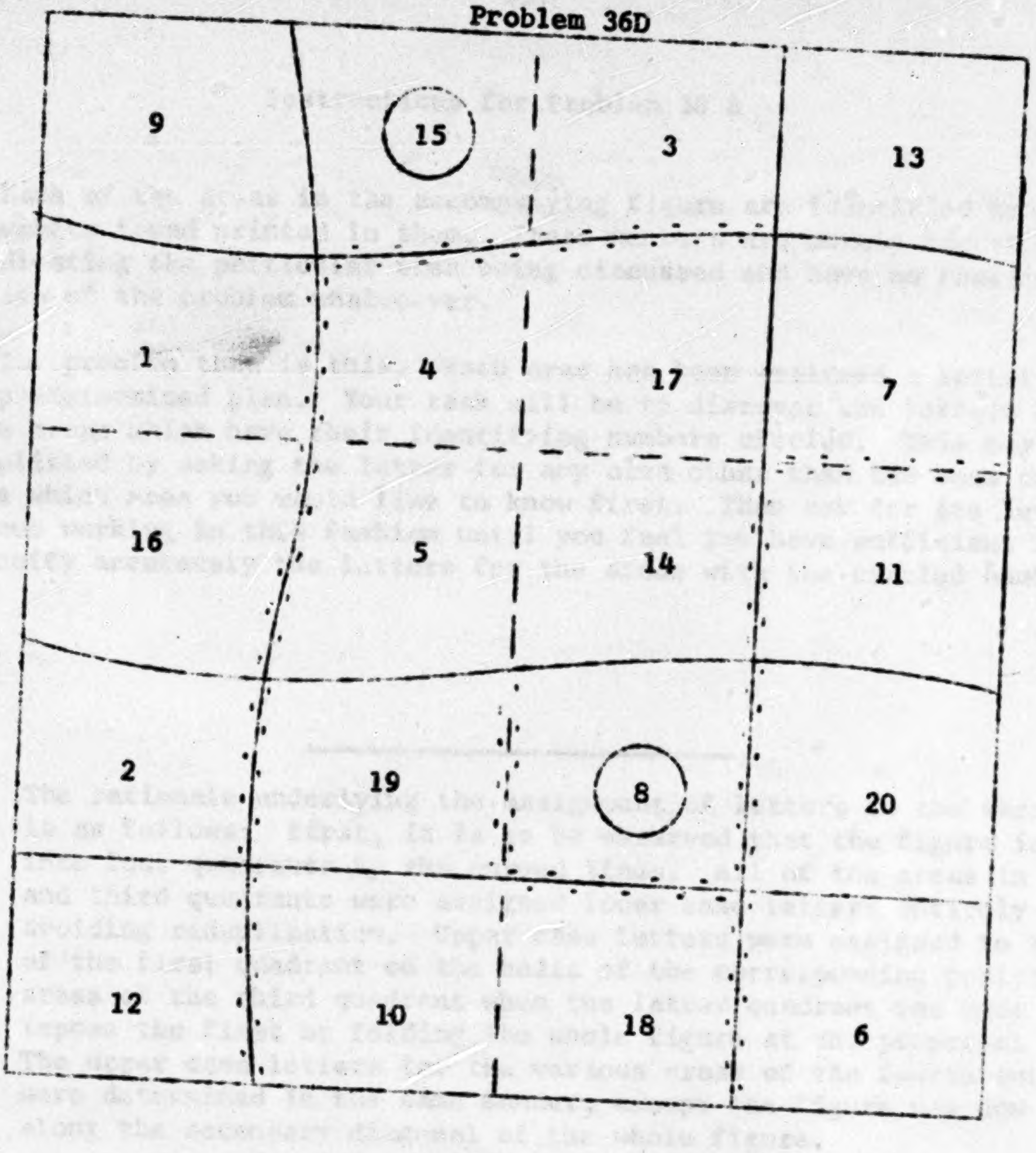


2a+2c	a+b+c+f	2a+b+f	3a+c
a+b+c+f	2b+2f	a+2b+f	2a+c+e
a+b+c+f	2b+c+f	2b+c+d	a+c+ d+e
2a+c+f	a+b+e+f	c+2d+e	2a+c+d
3a+f	2a+b+f	a+b+2d	3a+d

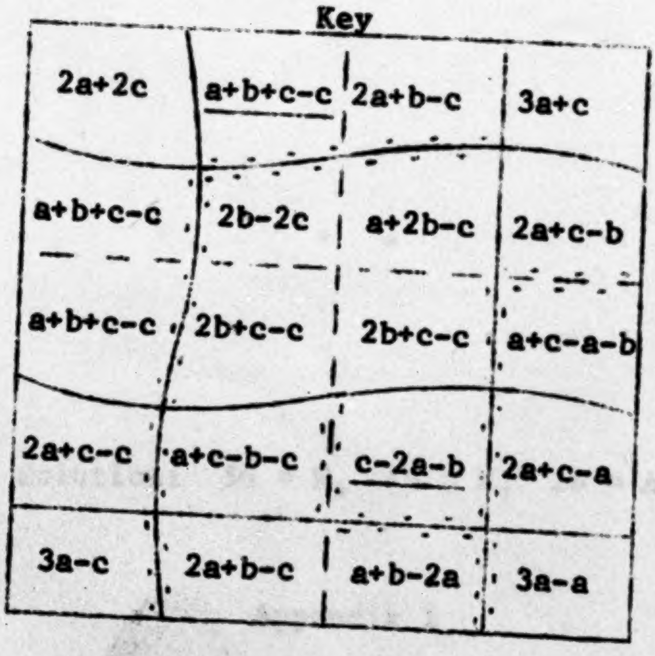
Note: lines or portions with dots along both sides appeared in red in the original.

Appendix I

Problem 36D


9
15
3
13
1
4
17
7
16
5
14
11
2
19
8
20
12
10
18
6

Key



2a+2c	a+b+c-c	2a+b-c	3a+c
a+b+c-c	2b-2c	a+2b-c	2a+c-b
a+b+c-c	2b+c-c	2b+c-c	a+c-a-b
2a+c-c	a+c-b-c	c-2a-b	2a+c-a
3a-c	2a+b-c	a+b-2a	3a-a

Note: Lines or portions of lines with dots along both sides appeared in red in the original.

Appendix I

Instructions for Problem 38 A

Each of the areas in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

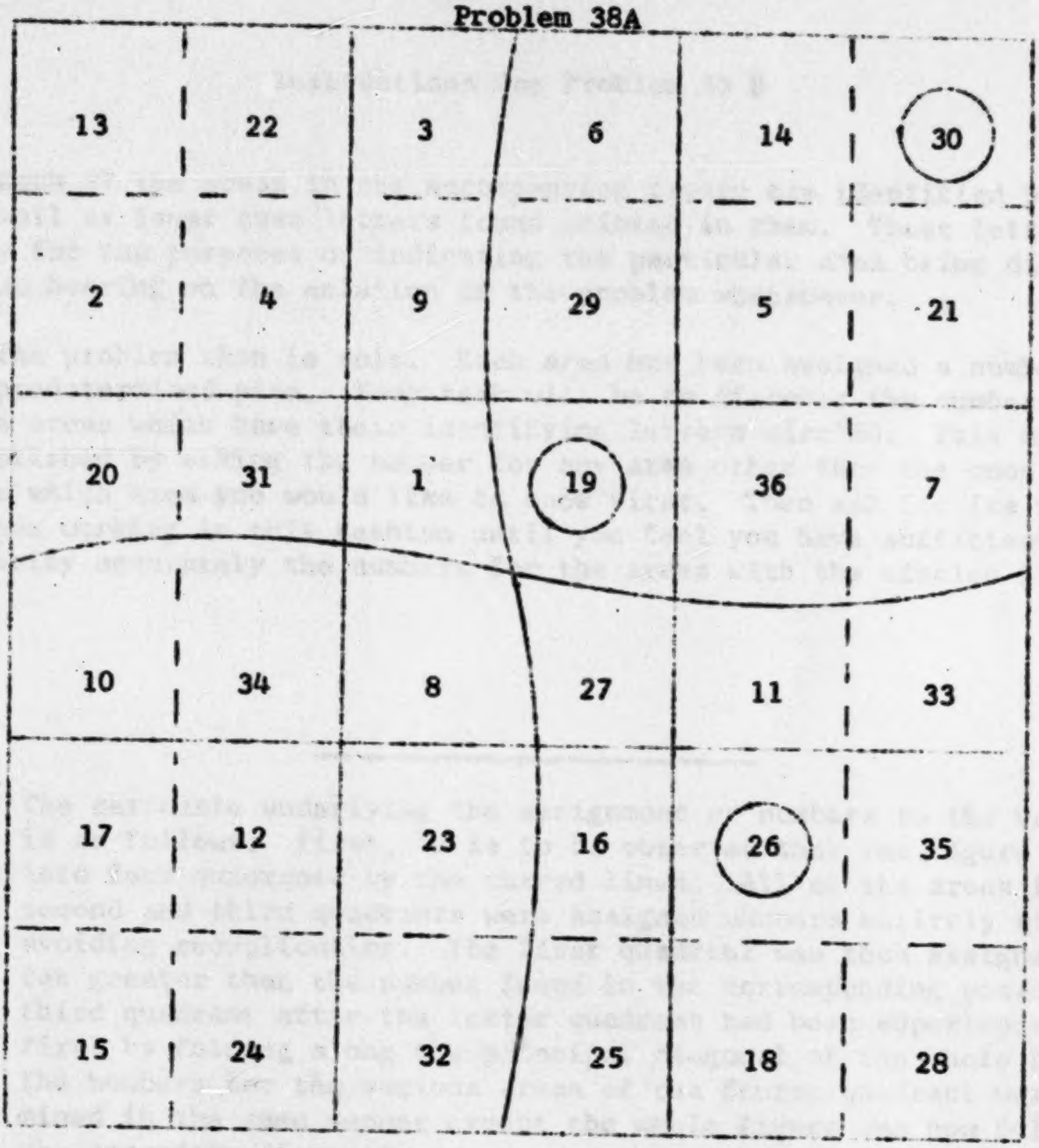
The problem then is this. Each area has been assigned a letter according to a predetermined plan. Your task will be to discover the letters belonging to the areas which have their identifying numbers circled. This may be accomplished by asking the letter for any area other than the ones circled. Decide which area you would like to know first. Then ask for its letter. Continue working in this fashion until you feel you have sufficient information to specify accurately the letters for the areas with the circled numbers.

Note: The rationale underlying the assignment of letters to the various areas is as follows: first, it is to be observed that the figure is divided into four quadrants by the curved lines. All of the areas in the second and third quadrants were assigned lower case letters entirely at random, avoiding reduplication. Upper case letters were assigned to the areas of the first quadrant on the basis of the corresponding position of the areas of the third quadrant when the latter quadrant was made to superimpose the first by folding the whole figure at the principal diagonal. The upper case letters for the various areas of the fourth quadrant were determined in the same manner, except the figure was now folded along the secondary diagonal of the whole figure.

Solution: 30 = N, 19 = M, 26 = A.

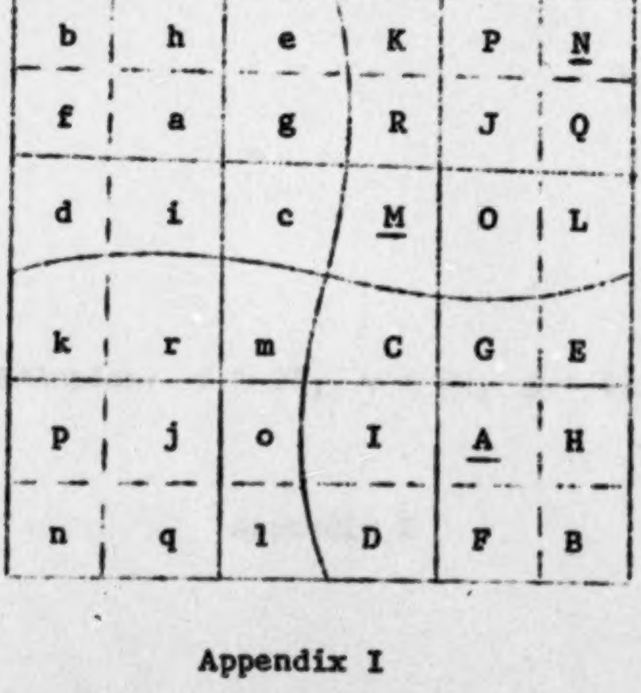
Appendix I

Problem 38A



13	22	3	6	14	30
2	4	9	29	5	21
20	31	1	19	36	7
10	34	8	27	11	33
17	12	23	16	26	35
15	24	32	25	18	28

Key



b	h	e	K	P	N
f	a	g	R	J	Q
d	i	c	M	O	L
k	r	m	C	G	E
p	j	o	I	A	H
n	q	l	D	F	B

Appendix I

Instructions for Problem 38 B

Each of the areas in the accompanying figure are identified by means of the small or lower case letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

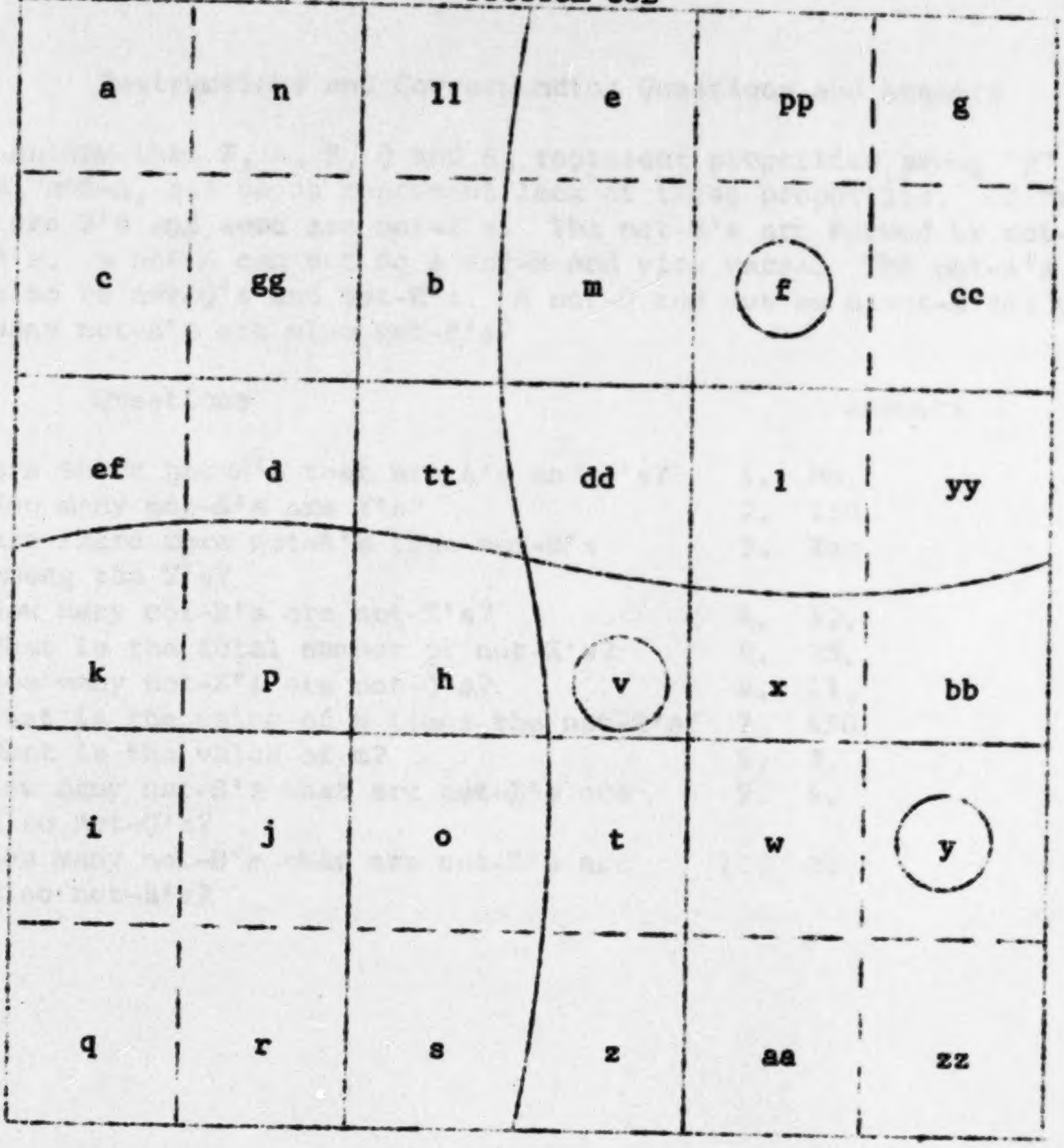
The problem then is this. Each area has been assigned a number according to a predetermined plan. Your task will be to discover the numbers belonging to the areas which have their identifying letters circled. This may be accomplished by asking the number for any area other than the ones circled. Decide which area you would like to know first. Then ask for its number. Continue working in this fashion until you feel you have sufficient information to specify accurately the numbers for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: first, it is to be observed that the figure is divided into four quadrants by the curved lines. All of the areas in the second and third quadrants were assigned numbers entirely at random, avoiding reduplication. The first quadrant was then assigned numbers ten greater than the number found in the corresponding position of the third quadrant after the latter quadrant had been superimposed on the first by folding along the principal diagonal of the whole figure. The numbers for the various areas of the fourth quadrant were determined in the same manner except the whole figure was now folded along the secondary diagonal.

Solution: f = 25, v = 26, y = 12.

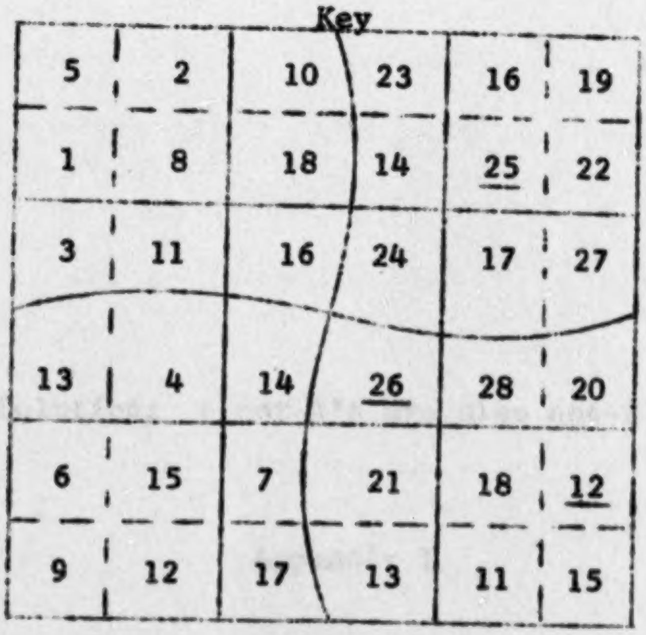
Appendix I

Problem 38B



a	n	ll	e	pp	g
c	gg	b	m	f	cc
ef	d	tt	dd	l	yy
k	p	h	v	x	bb
i	j	o	t	w	y
q	r	s	z	aa	zz

Key



5	2	10	23	16	19
1	8	18	14	25	22
3	11	16	24	17	27
13	4	14	26	28	20
6	15	7	21	18	12
9	12	17	13	11	15

Appendix I

Problem 31 C'

Instructions and Corresponding Questions and Answers

Assume that Z, A, B, Q and R, represent properties among "y" objects. Not-Z, not-A, and so on represent lack of these properties. Of "y" objects some are Z's and some are not-Z's. The not-Z's are formed by not-A's and not-B's. A not-A can not be a not-B and vice versa. The not-A's and not-B's may also be not-Q's and not-R's. A not-Q can not be a not-R and vice versa. How many not-A's are also not-R's?

Questions	Answers
1. Are there not-Z's that are A's and Q's?	1. No.
2. How many not-A's are Y's?	2. 150.
3. Are there more not-A's than not-B's among the Y's?	3. Yes.
4. How many not-B's are not-Z's?	4. 12.
5. What is the total number of not-Z's?	5. 25.
6. How many not-Z's are not-Q's?	6. 11.
7. What is the value of m times the not-R's?	7. 450.
8. What is the value of m?	8. 3.
9. How many not-B's that are not-Z's are also not-Q's?	9. 4.
10. How many not-B's that are not-Z's are also not-R's?	10. 8.

Solution: 6 not-A's are also not-R's.

Problem 35 A'

Instructions and Corresponding Questions and Answers

The athletic director at St. Lawrence High School was asked by the principal of the school to report on the academic standing of the fellows involved in the sports program. Only a limited amount of information was available to the director, but it was sufficient to provide the principal with the required information.

The director had to give the number of C students that were on the basketball team. What was that number? Assume that no player is a member of more than one team.

Questions	Answers
1. What are the sports in which St. Lawrence High School participates?	1. Football, basketball, and track.
2. Could any fellow with less than a C average be in the sports program?	2. No.
3. How many C students are there in this high school?	3. 838.
4. How many A students are there in the sports program?	4. 14.
5. How many B students play on the basketball team?	5. 4.
6. What is the total number of students in the sports program?	6. 79.
7. How many sophomores are in the sports program?	7. 22.
8. How many B students are in the sports program?	8. 26.
9. How many C students are on the football team?	9. 18.
10. How many A students are there in the school?	10. 280.
11. How many A students are on the football team?	11. 5.
12. How many men are on the track team?	12. 28.
13. How many B students are on the football team?	13. 12.
14. How many fellows made the All-City team in their respective sport?	14. 6.
15. How many A students are on the track team?	15. 6.
16. How many men are on the football team?	16. 35.
17. How many C students are on the track team?	17. 12.

Appendix I

Problem 35 A'
(Continued)

Questions	Answers
18. How many A students are on the basketball team?	18. 3.
19. How many B students are on the track team?	19. 10.

Solution: 9 C students.

Appendix I

Instructions for Problem 32 E

Each of the areas in the accompanying figure are identified by means of the letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

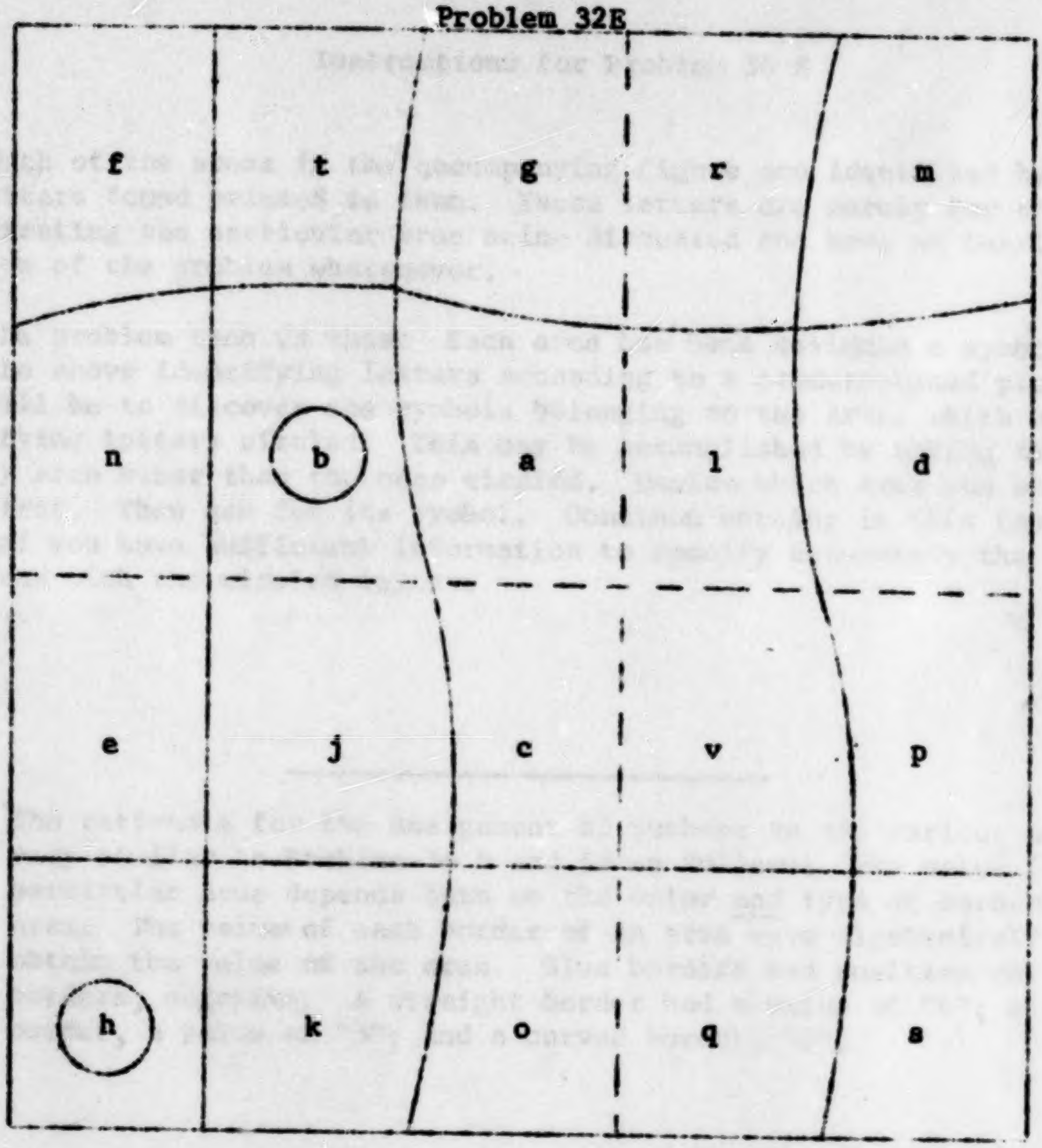
The problem then is this. Each area has been assigned a symbol other than the above identifying letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying letters circled. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: beginning with area "f" and proceeding from left to right by successive rows, numbers were assigned the areas according to their relative position in a sequence of numbers formed by an alternating series of first three positive and then three negative in the order of increasing absolute magnitude. Each series increases in absolute magnitude independently of the other, though, of course, undergoing interruption.. The positive series begins with "2" and the negative, with a "-3".

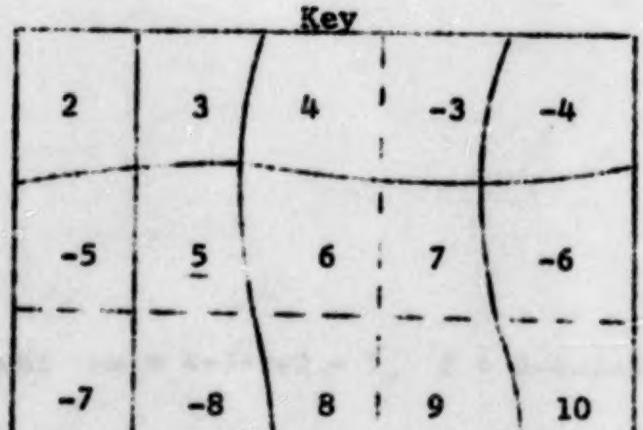
Solution: b = 5, h = -9.

Appendix I

Problem 32E


f
t
g
r
m
n
b
a
l
d
e
j
c
v
p
h
k
o
q
s

Key


2
3
4
-3
-4
-5
5
6
7
-6
-7
-8
8
9
10
-9
-10
-11
11
12

Instructions for Problem 36 E

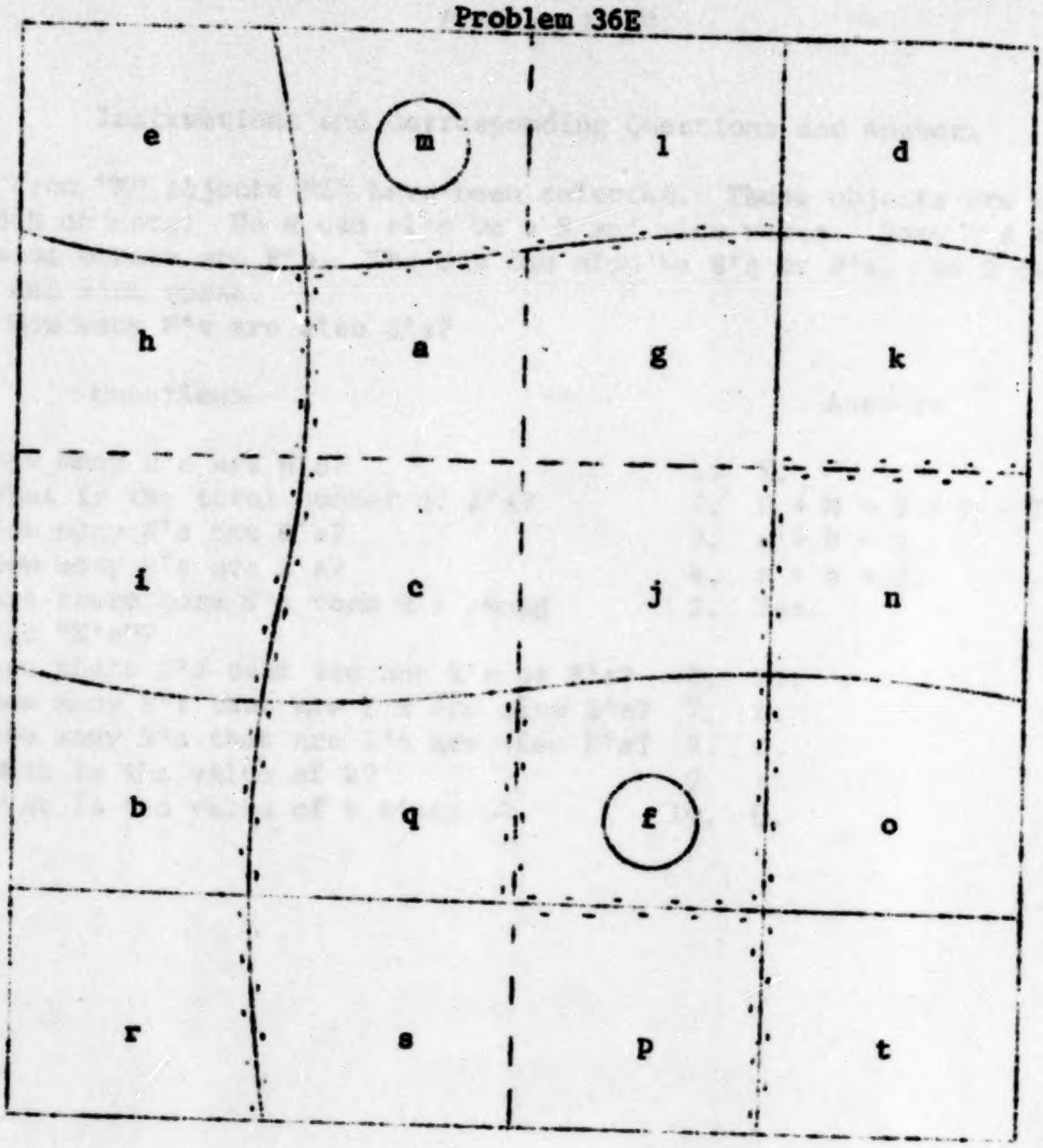
Each of the areas in the accompanying figure are identified by means of the letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a symbol other than the above identifying letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying letters circled. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

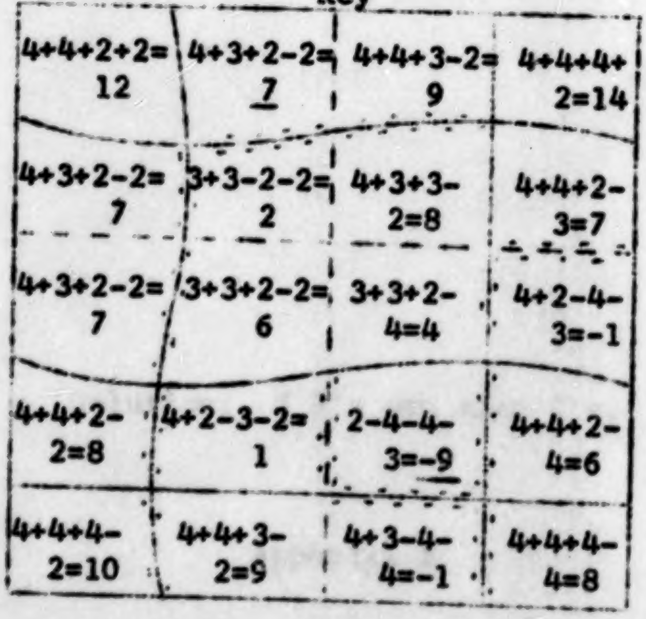
Note: The rationale for the assignment of numbers to the various areas is very similar to Problem 36 D and is as follows: the value for a particular area depends both on the color and type of borders of the area. The values of each border of an area were algebraically summed to obtain the value of the area. Blue borders had positive values; red borders, negative. A straight border had a value of "4"; a dotted border, a value of "3"; and a curved border, "2".

Solution: -m = 4+3+2-2 = 7, f = 2-4-4-3 = -9

Problem 36E



Key



4+4+2+2= 12	4+3+2-2= 7	4+4+3-2= 9	4+4+4+ 2=14
4+3+2-2= 7	3+3-2-2= 2	4+3+3- 2=8	4+4+2- 3=7
4+3+2-2= 7	3+3+2-2= 6	3+3+2- 4=4	4+2-4- 3=-1
4+4+2- 2=8	4+2-3-2= 1	2-4-4- 3=-9	4+4+2- 4=6
4+4+4- 2=10	4+4+3- 2=9	4+3-4- 4=-1	4+4+4- 4=8

Note: Lines or portions of lines with dots along both sides appeared in red in the original.

Appendix I

Problem 31 D'

Instructions and Corresponding Questions and Answers

From "X" objects "Z" have been selected. These objects are composed of R and S objects. No R can also be a S and vice versa. Some R's are also E's and some others are F's. The S's can also be E's or F's. No E can also be an F and vice versa.

How many F's are also S's?

Questions	Answers
1. How many R's are W's?	1. Y.
2. What is the total number of Z's?	2. $H + M = J + P = T$.
3. How many Z's are E's?	3. $a + b = H$.
4. How many R's are Z's?	4. $a + c = J$.
5. Are there more S's than R's among the "X's"?	5. Yes.
6. Are there Z's that are not R's or S's?	6. No.
7. How many S's that are Z's are also E's?	7. b.
8. How many R's that are Z's are also F's?	8. c.
9. What is the value of k?	9. r.
10. What is the value of k times L?	10. Q.

Solution: d F's are also S's.

Problem 35 B'

Instructions and Corresponding Questions and Answers

"A" objects belong either to X, Y or Z types. Each of these types in turn may be either an M, an N or an O. From the questions and answers you may discover the various relationships between the objects. Make use of the available information to determine how many of the "A" objects are both Y and O.

Questions	Answers
1. How many S's are T's?	1. 200.
2. How many M's are there among the A's?	2. 22.
3. How many G's are there among the A's?	3. None.
4. How many N's are also Y's?	4. 5.
5. What is the total number of "A" objects?	5. 72.
6. How many Z's are also A's?	6. 31.
7. How many M's are also X's?	7. 4.
8. How many N's are also A's?	8. 23.
9. How many N's are also X's?	9. 7.
10. How many M's are also T's?	10. 300.
11. How many O's are X's?	11. 6.
12. How many N's are also Z's?	12. 11.
13. How many M's are also Y's?	13. 10.
14. How many O's are also Z's?	14. 12.
15. How many X's are there among the A's?	15. 17.
16. How many M's are also Z's?	16. 8.
17. How many H's are there among the T's?	17. 2.

Solution: 9 of the "A" objects are both Y and O.

Instructions for Problem 32 F

Each of the areas in the accompanying figure are identified by means of the letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

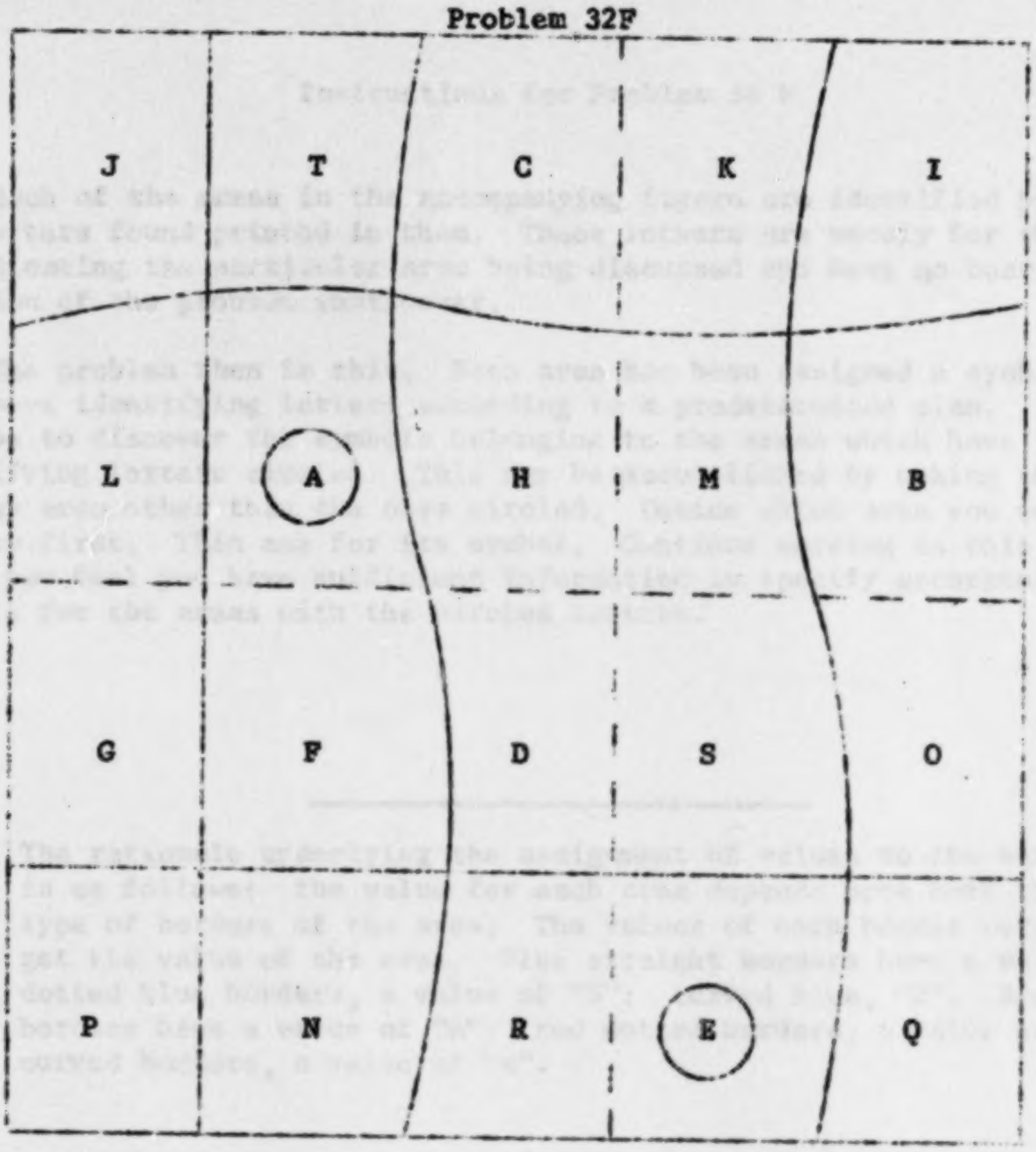
The problem then is this. Each area has been assigned a symbol other than the above identifying letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying letters circled. This may be accomplished by asking the symbols for any area other then the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

Note: The rationale underlying the assignment of numbers to the various areas is as follows: beginning with area "J" and proceeding downward by successive columns, numbers were assigned the areas according to their relative position in a sequence of numbers formed by an alternating series of first three positive and then three negative numbers in the order of increasing absolute magnitude. Each series increases in absolute magnitude independently of the other, though, of course, undergoing interruption. The positive series begins with "2" and the negative, with "-3".

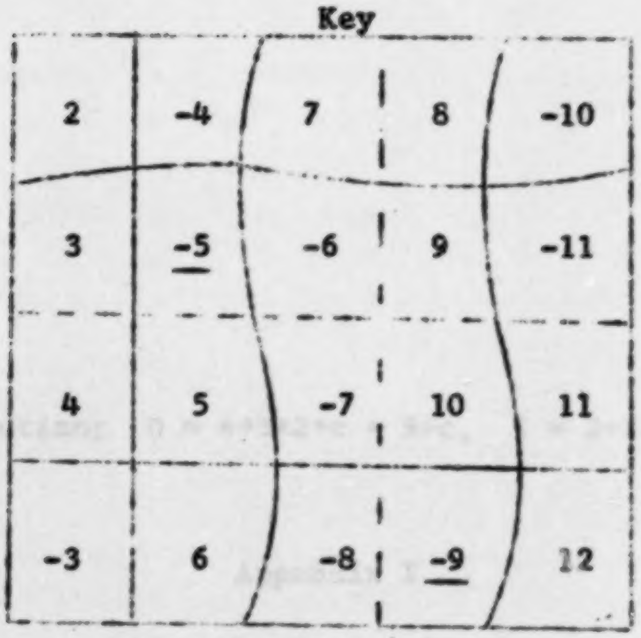
Solution: A = -5, E = -9

Appendix I

Problem 32F


J
T
C
K
I
L
A
H
M
B
G
F
D
S
O
P
N
R
E
Q

Key


2
-4
7
8
-10
3
-5
-6
9
-11
4
5
-7
10
11
-3
6
-8
-9
12

Appendix I

Instructions for Problem 36 F

Each of the areas in the accompanying figure are identified by means of the letters found printed in them. These letters are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

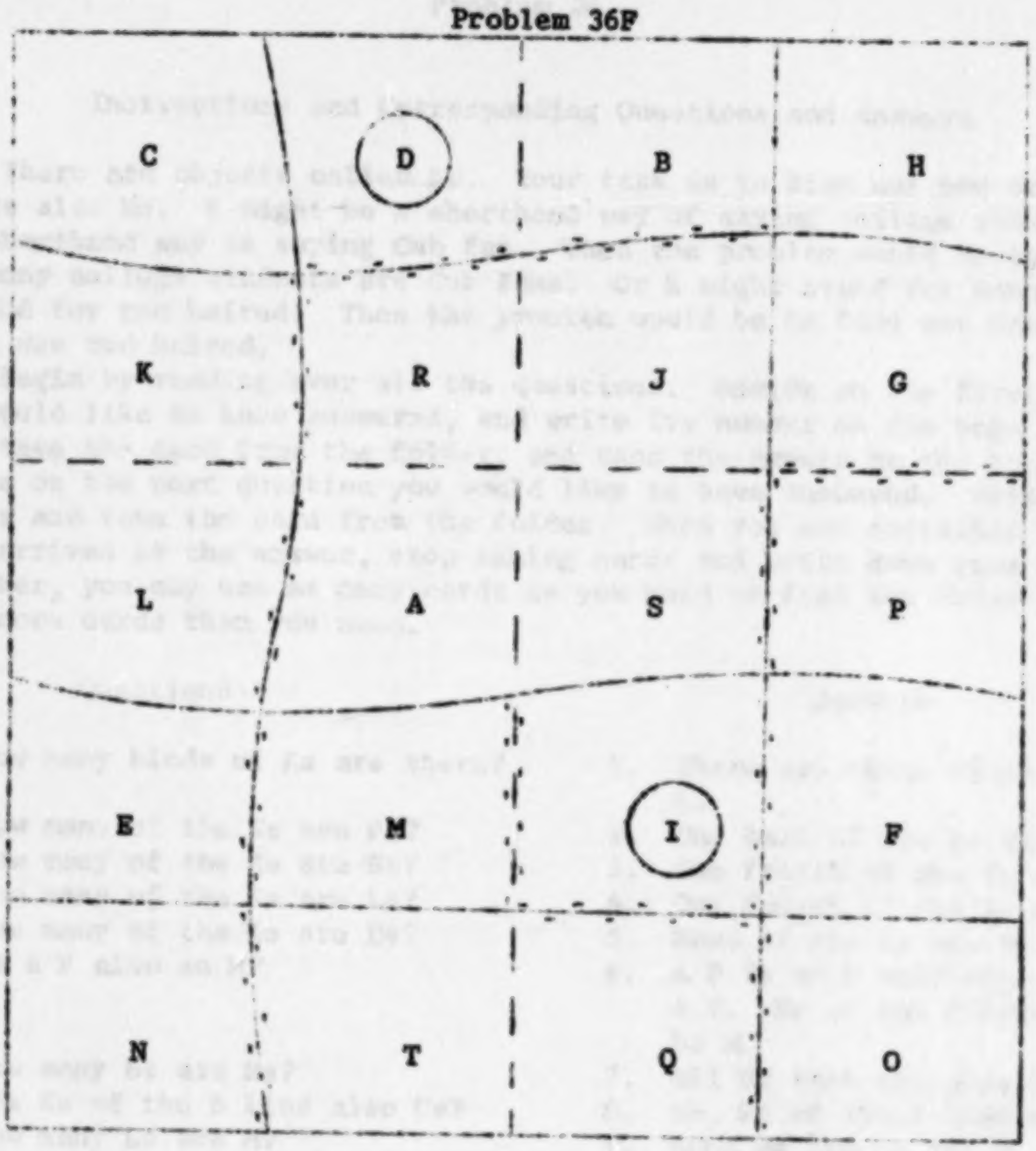
The problem then is this. Each area has been assigned a symbol other than the above identifying letters according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying letters circled. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled letters.

Note: The rationale underlying the assignment of values to the various areas is as follows: the value for each area depends upon both the color and type of borders of the area. The values of each border were summed to get the value of the area. Blue straight borders have a value of "4"; dotted blue borders, a value of "3"; curved blue, "2". Red straight borders have a value of "a"; red dotted borders, a value of "b"; red curved borders, a value of "c".

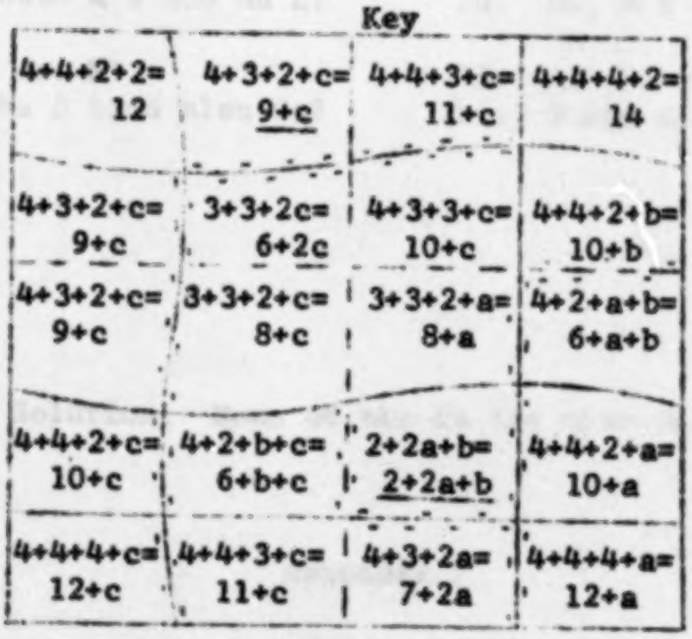
Solution: $D = 4+3+2+c = 9+c$, $I = 2+2a+b$.

Appendix I

Problem 36F



Key



4+4+2+2= 12	4+3+2+c= 9+c	4+4+3+c= 11+c	4+4+4+2= 14
4+3+2+c= 9+c	3+3+2c= 6+2c	4+3+3+c= 10+c	4+4+2+b= 10+b
4+3+2+c= 9+c	3+3+2+c= 8+c	3+3+2+a= 8+a	4+2+a+b= 6+a+b
4+4+2+c= 10+c	4+2+b+c= 6+b+c	2+2a+b= 2+2a+b	4+4+2+a= 10+a
4+4+4+c= 12+c	4+4+3+c= 11+c	4+3+2a= 7+2a	4+4+4+a= 12+a

Note: lines or portions of lines with dots along both sides appeared in red in the original.

Appendix I

Problem 26

Instructions and Corresponding Questions and Answers

There are objects called Ks. Your task is to find out how many of the Ks are also Ms. K might be a shorthand way of saying college student, and M a shorthand way os saying Cub fan. Then the problem would be to find out how many college students are Cub fans. Or K might stand for Democrat, and M stand for red haired. Then the problem would be to find out how many Democrats are red haired.

Begin by reading over all the questions. Decide on the first question you would like to have answered, and write its number on the page provided. Then take the card from the folder, and read the answer on the back. Then decide on the next question you would like to have answered. Write down its number and take the card from the folder. When you are satisfied that you have arrived at the answer, stop taking cards and write down your answer. Remember, you may use as many cards as you need to find the answer, but don't draw more cards than you need.

Questions	Answers
1. How many kinds of Ks are there?	1. There are three kinds: P, B and L.
2. How many of the Ks are Ps?	2. One half of the Ks are Ps.
3. How many of the Ks are Bs?	3. One fourth of the Ks are Bs.
4. How many of the Ks are Ls?	4. One fourth of the Ks are Ls.
5. How many of the Ks are Ds?	5. None of the Ks are Ds.
6. Is a P also an M?	6. A P is an M only when it is not a K. Ks of the P kind cannot be M.
7. How many Bs are Ms?	7. All Bs that are also Cs are Ms.
8. Are Ks of the B kind also Cs?	8. No, Ks of the B kind are not Cs.
9. How many Ls are M?	9. None of the Ls are M.
10. May a K be both a P and a B?	10. No, a K cannot be both a P and a B.
11. May a K be both a P and an L?	11. No, a K cannot be both a P and an L.
12. May a K be both a B and an L?	12. No, a K cannot be both a B and an L.
13. How many Ds are M?	13. All Ds that are not Cs are M.
14. Are Ks of the D kind also Cs?	14. There are no Ks of the D kind.

Solution: None of the Ks are also Ms.

Appendix I

Problem 41 A

Instructions and Corresponding Questions and Answers

At a textile factory in a small town the telephone system connecting the various factory offices is rather primitive. However, communication between offices is greatly aided by the network, since the offices are considerably separated.

Basically, the factory has a north wing comprising offices A, B and C, and a south wing in which are offices D, E, F, G and H. At the time the phones were installed only seven were available and consequently office G is without one. Because of the way in which the phones have been wired certain limitations have been placed on their use. The limitations are these:

1) There is not a single phone from which all the other offices with phones may be called.

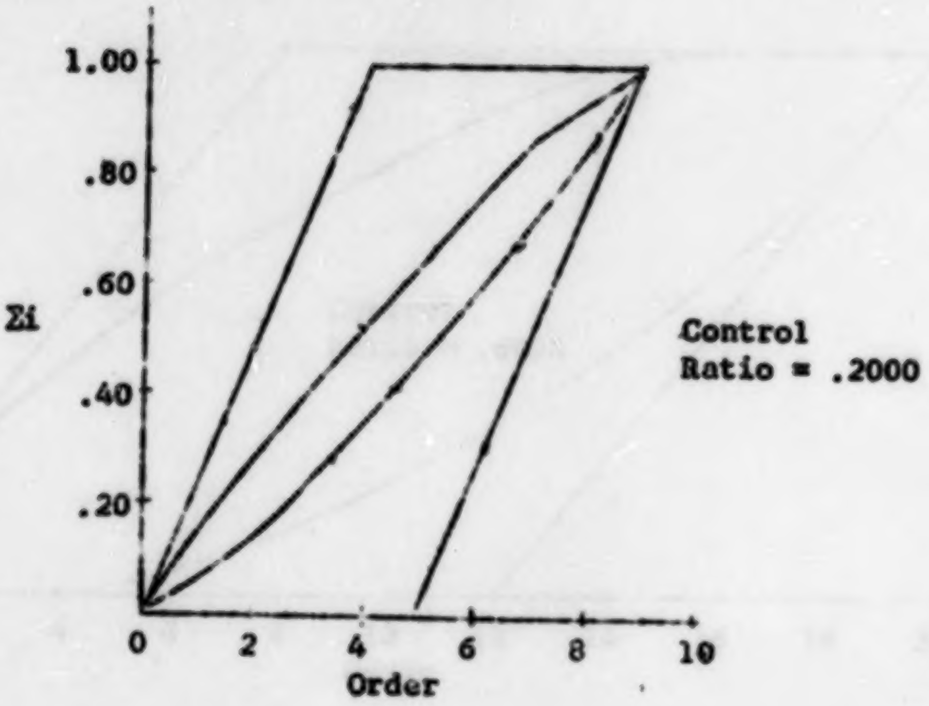
2) The fact that one office can call another does not necessarily mean that the last office can call the first.

With this present confusion in the telephone network a worker in office B would like to contact someone in office H. What is the most efficient way in which this can be accomplished.

Questions	Answers
1. Is there a line from any office in the north wing to H?	1. No.
2. Which office can issue outgoing calls to the greatest number of other offices?	2. Office E.
3. What offices can office C call?	3. Offices D & E.
4. Can office A call office H?	4. No.
5. From which offices in the south wing can office H receive calls?	5. Only from office D.
6. What offices in the north wing can call offices in the south wing?	6. Only office C.
7. Can every office in the north wing call every office in the south wing?	7. No.
8. Can office B call office A?	8. Yes.
9. What offices in the north wing can call office C?	9. Only office B.
10. Can office C call office B?	10. No.
11. What offices can office H call?	11. None.
12. Can every office in the south wing call every office in the north wing?	12. No.
13. What offices can office E call?	13. Offices A, B, C, D, and F.
14. Can office B call office H?	14. No.
15. Can any office in the south wing call any office in the north wing?	15. Yes, only office E.

Solution: B→C→D→H.

Appendix I



Chi Square = 1.71 (A Kolmogorov-Smirnov application of the Chi Square).

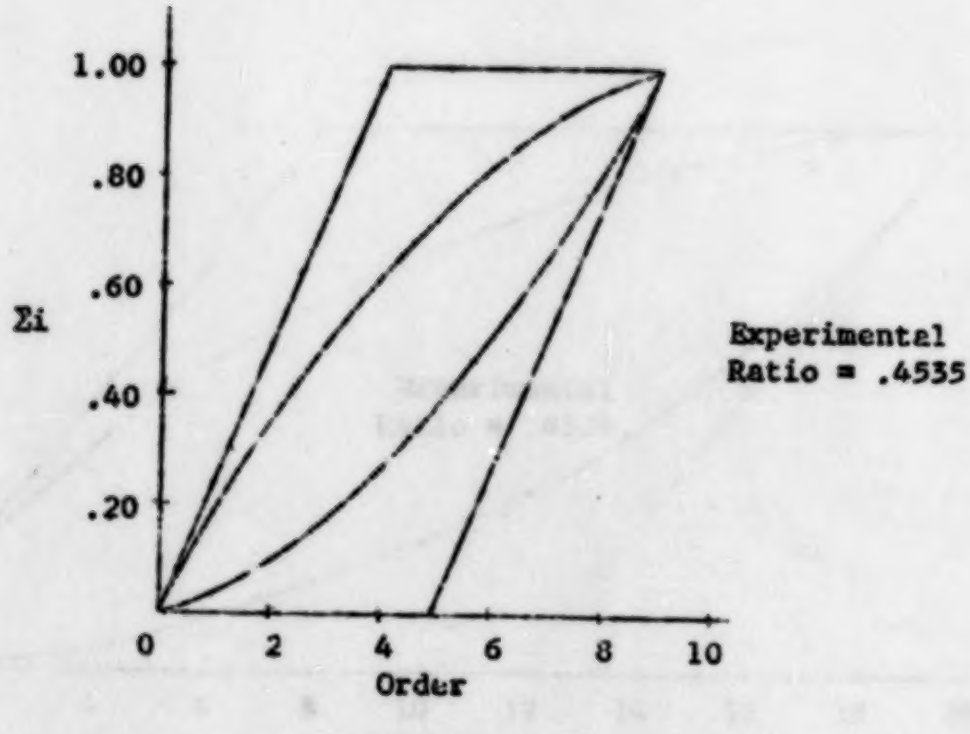
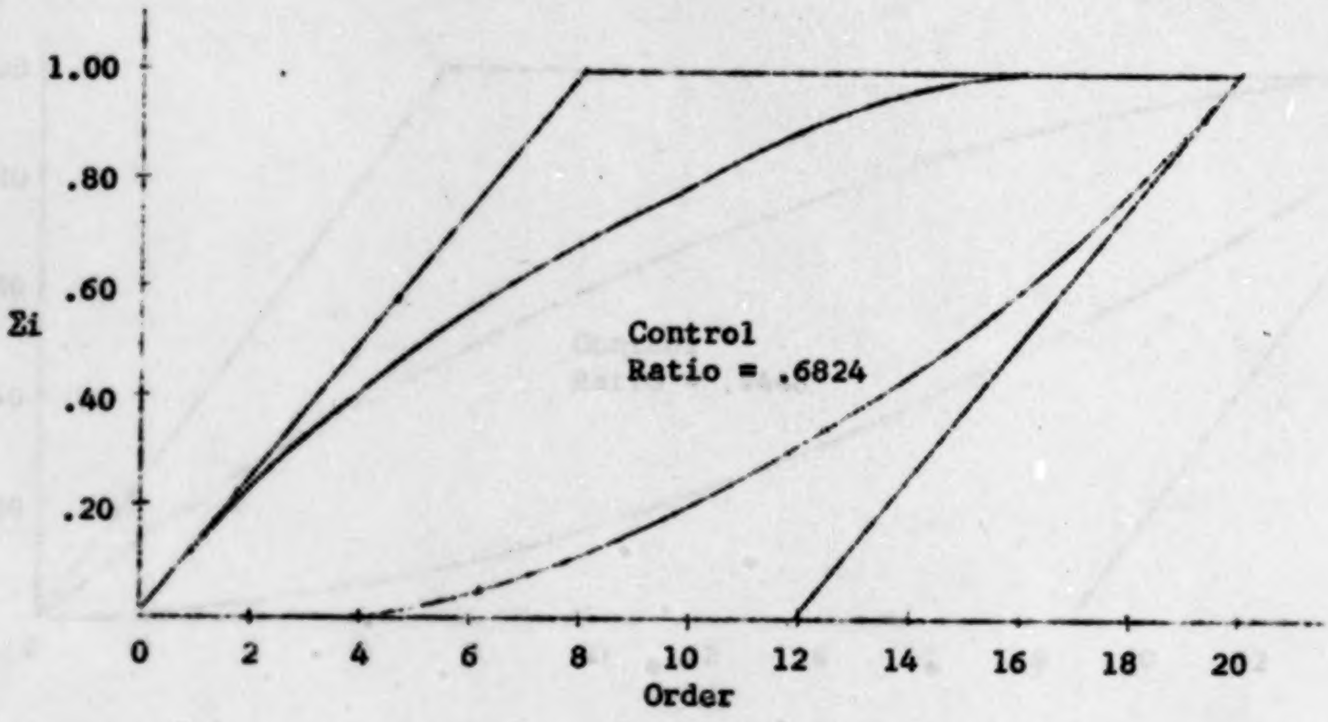


Figure 1. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Pre-Training Problem 1 and Values of Ratios.

Appendix II



Chi Square = 1.28

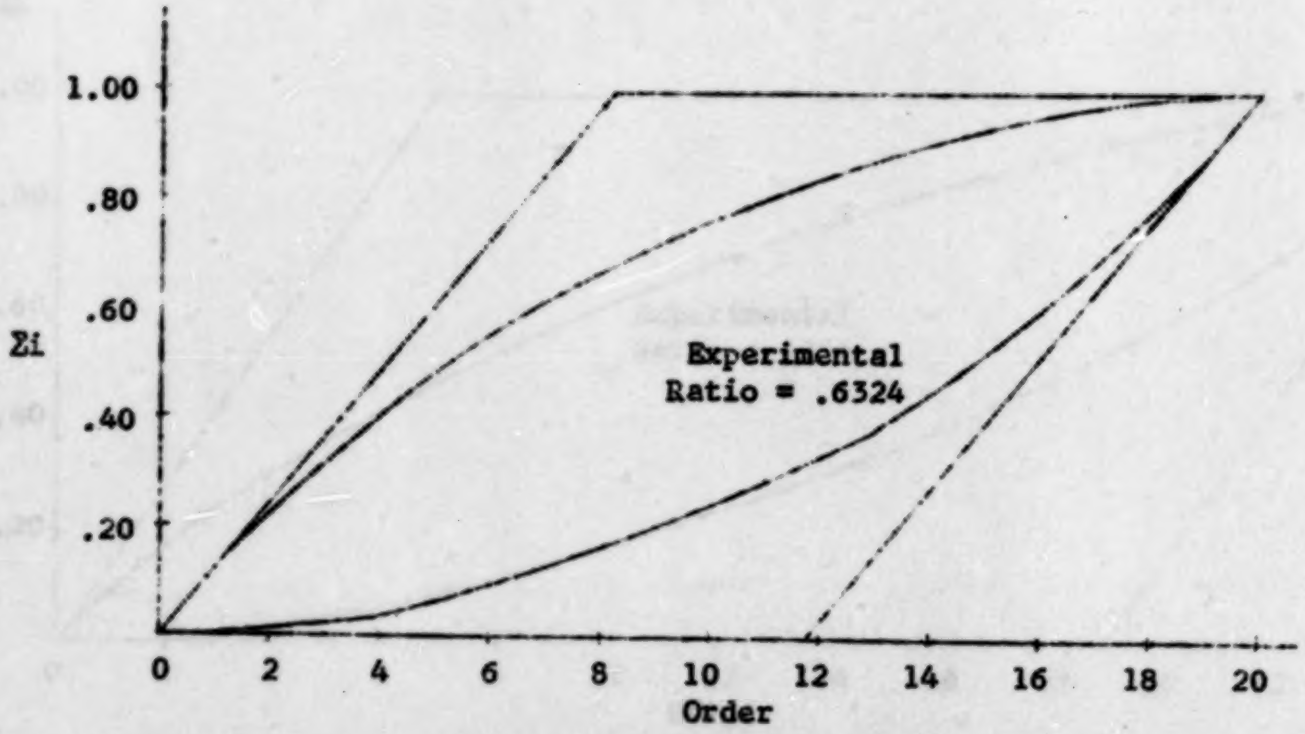
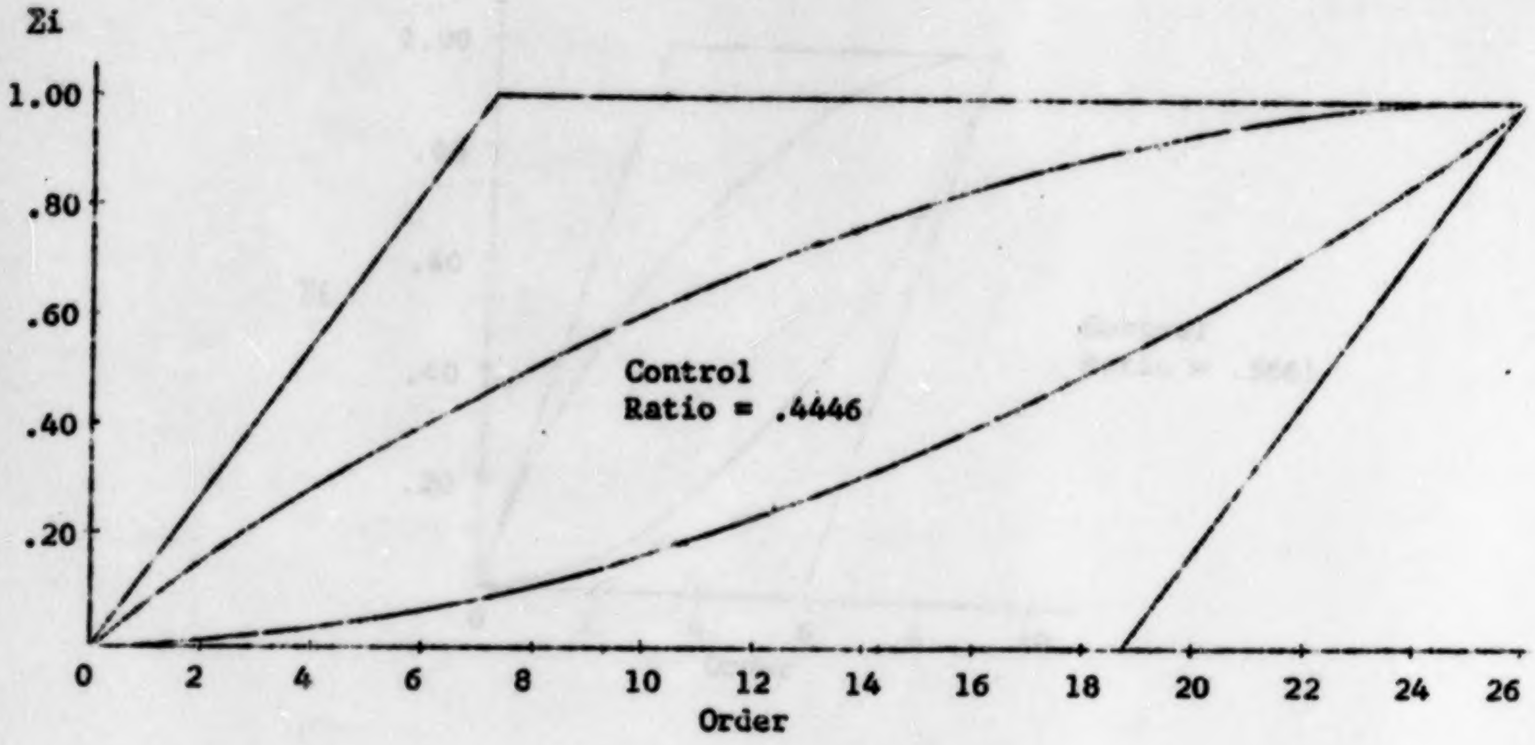


Figure 2. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Pre-Training Problem 19 and Values of Ratios.

Appendix II



Chi Square = .268

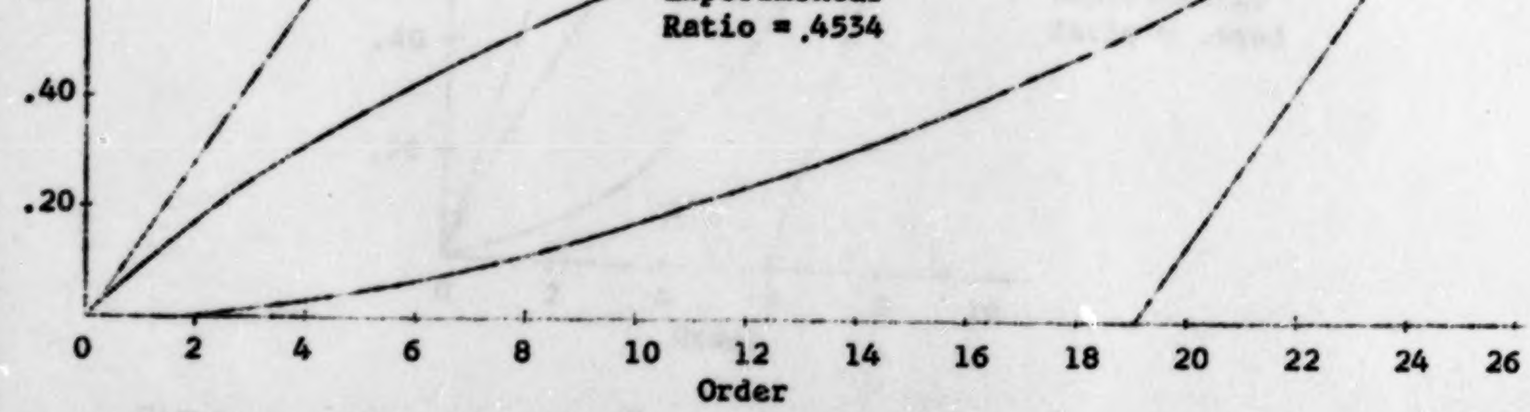
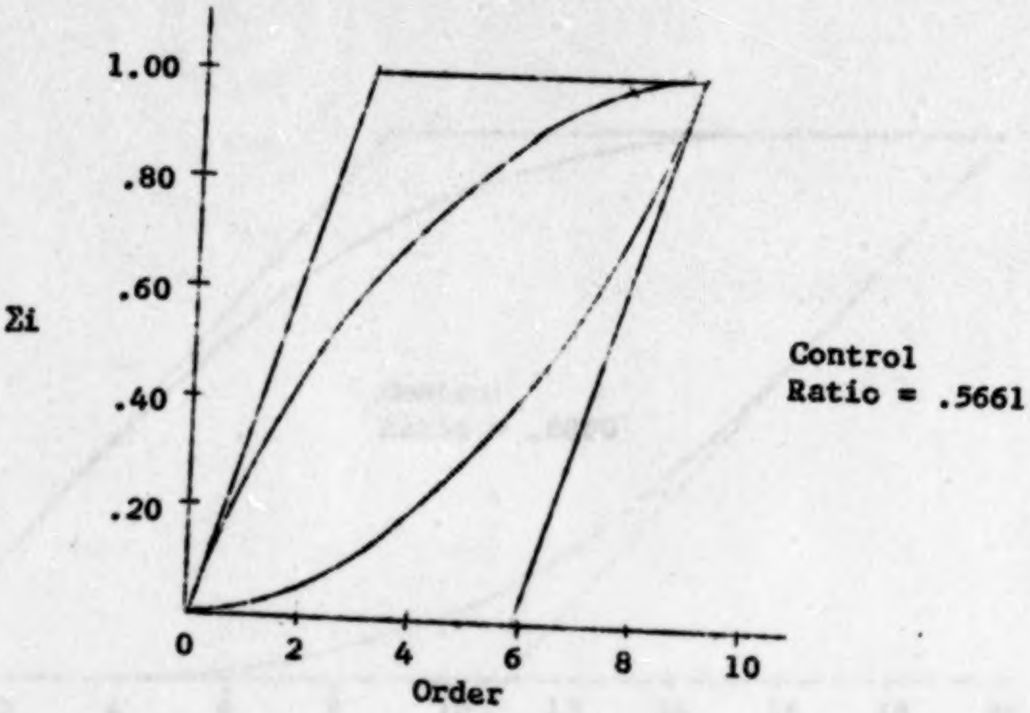


Figure 3. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Pre-Training Problem 25 and Values of Ratios.

Appendix II



Chi Square = .0598

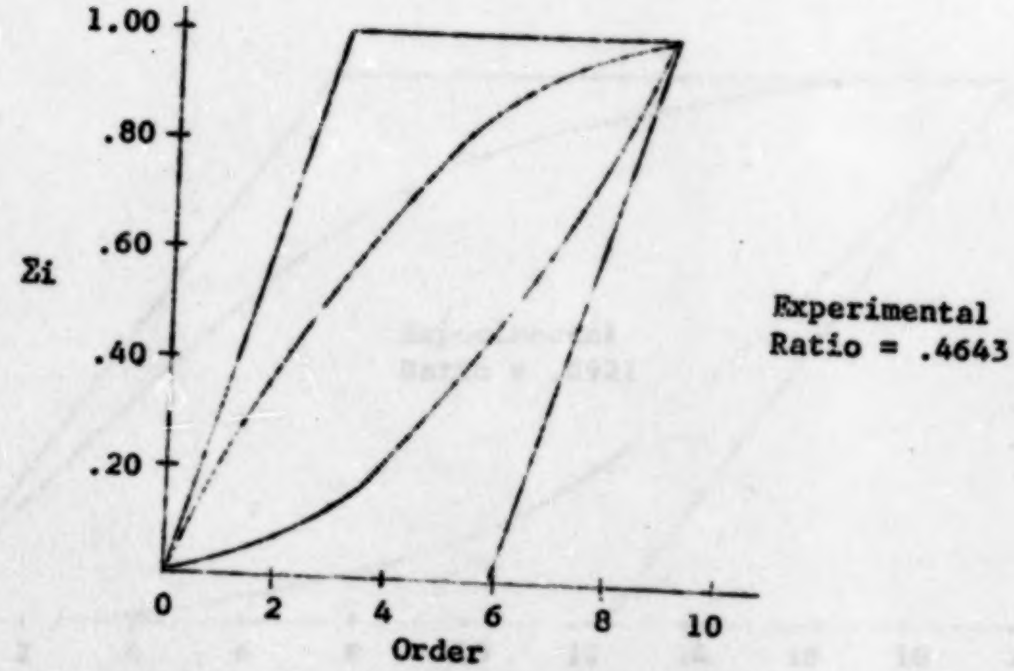
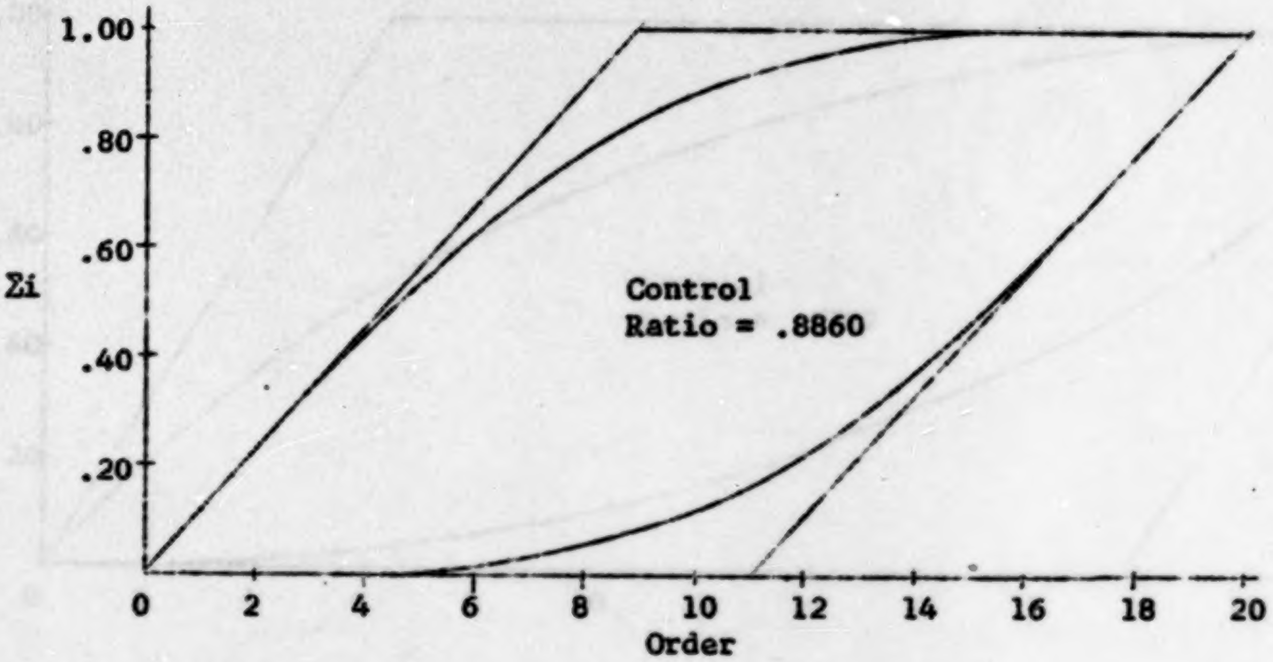


Figure 4. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Pre-Training Problem 1 and Values of Ratios.

Appendix II



Chi Square = .6392

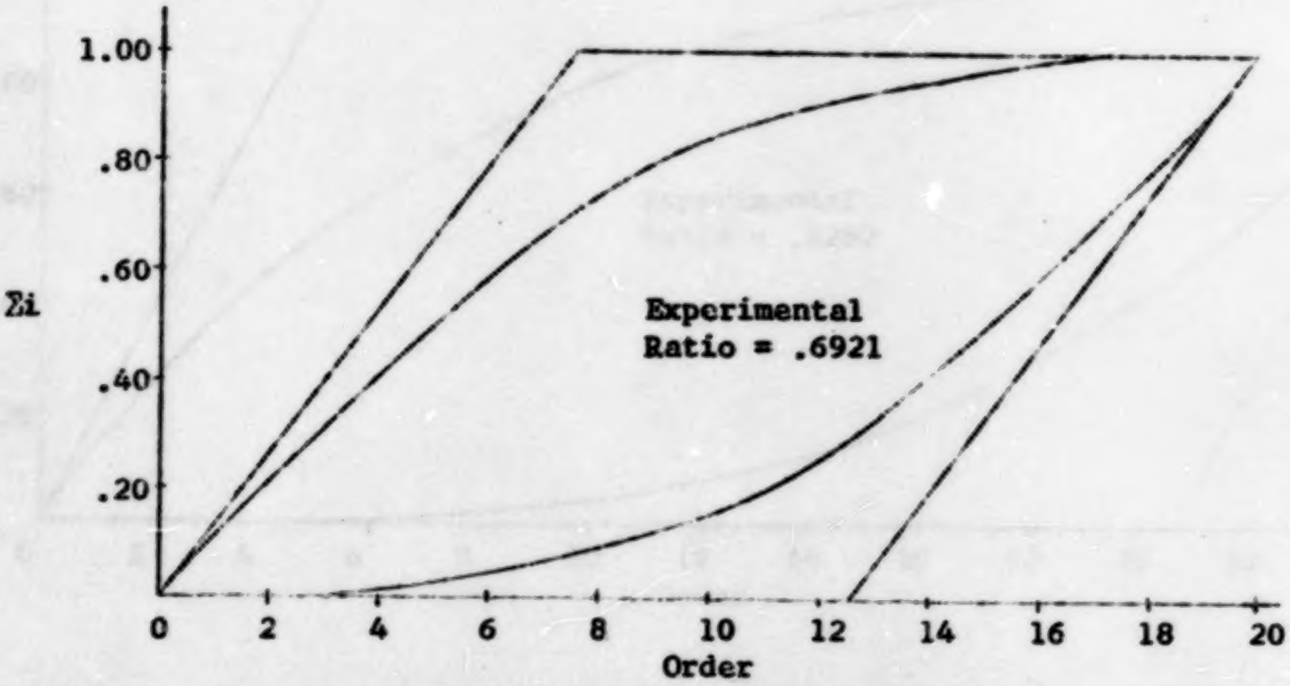
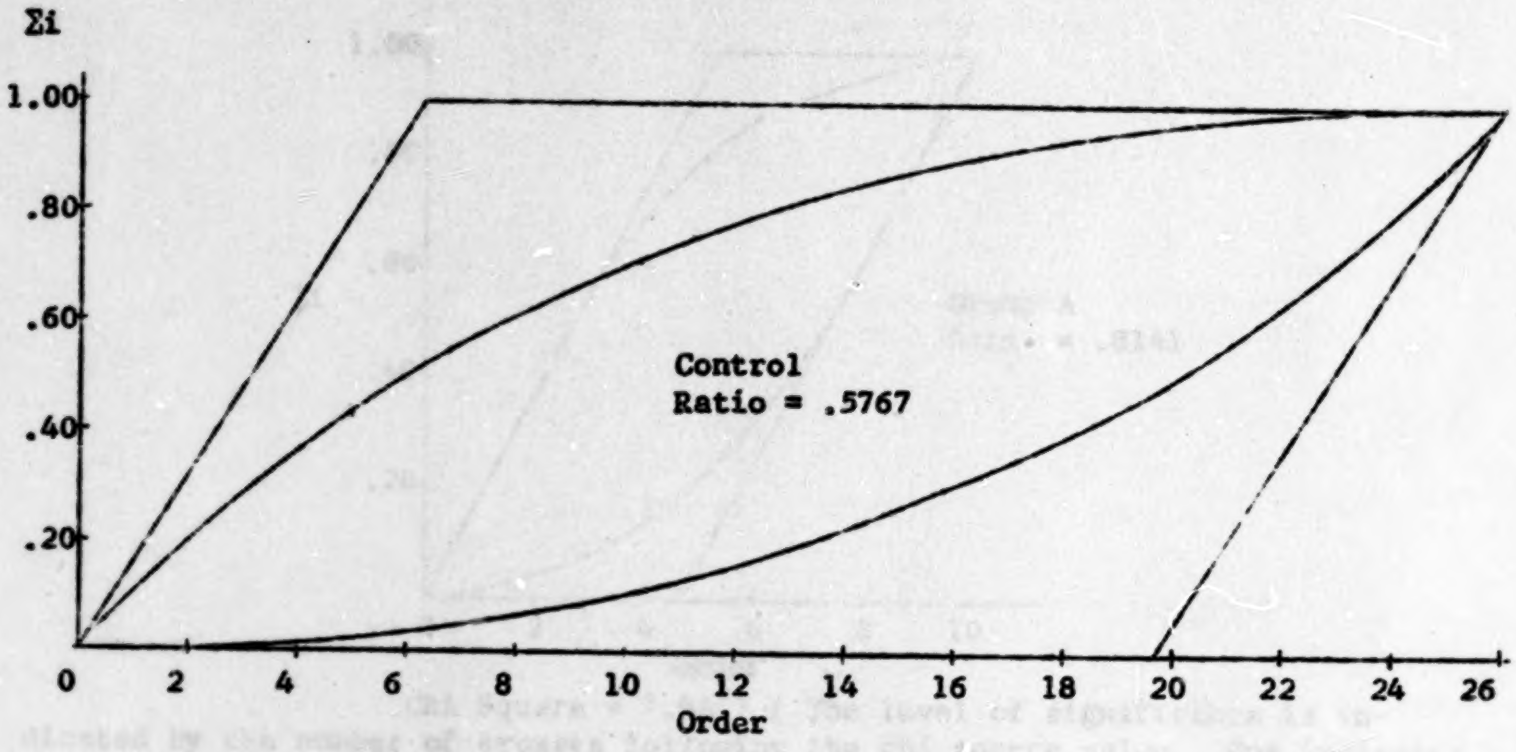


Figure 5. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Pre-Training Problem 19 and Values of Ratios.

Appendix II



Chi Square = 3.0654

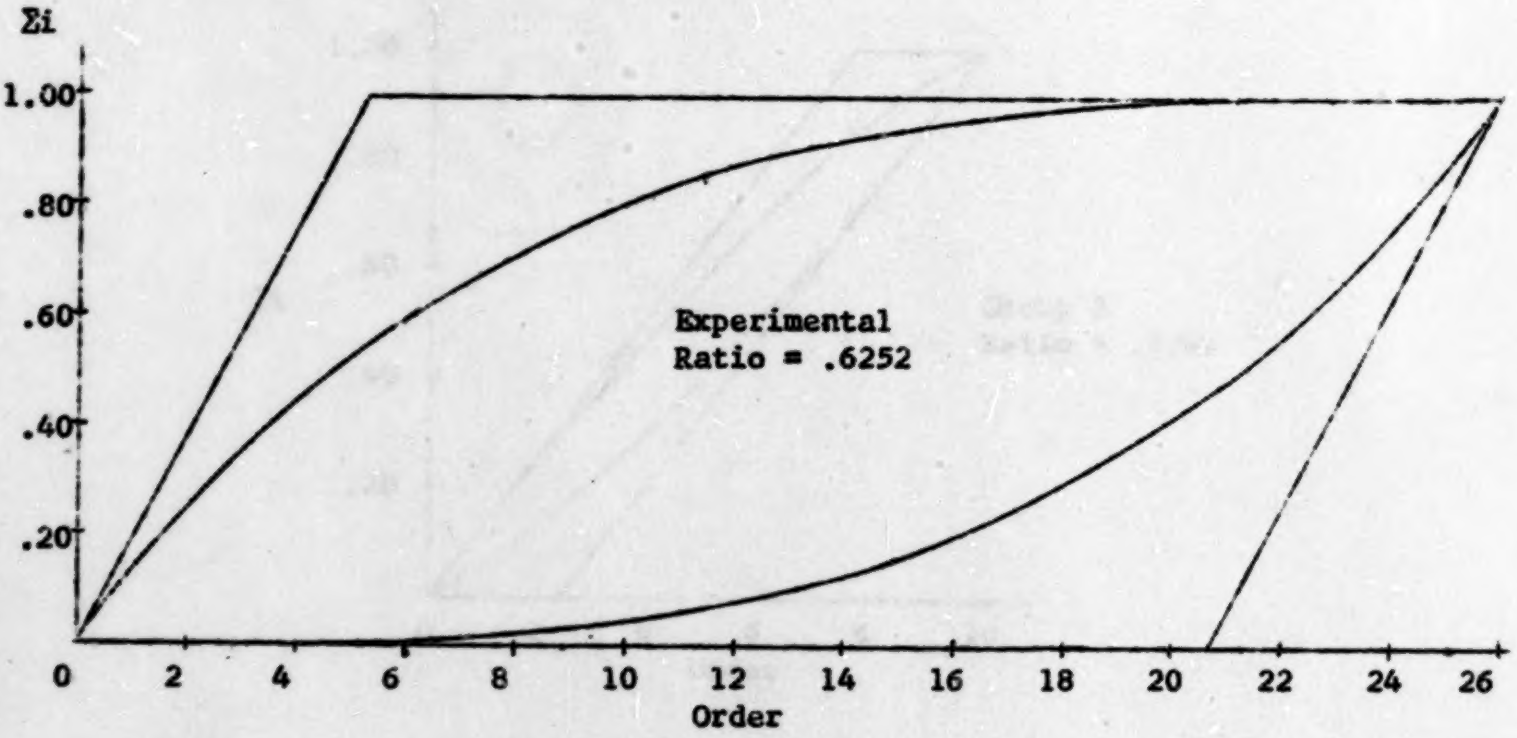
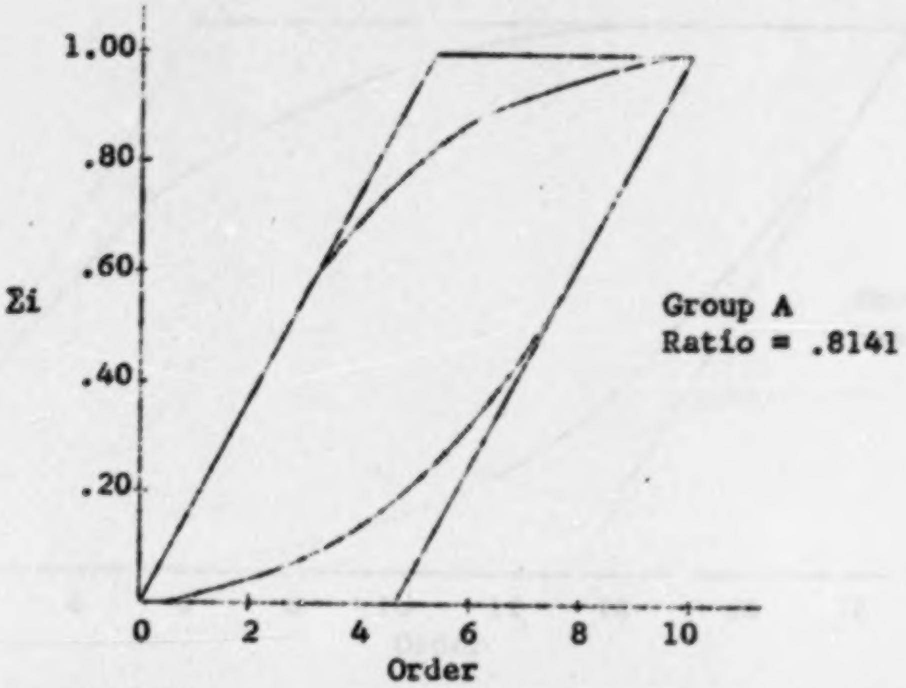


Figure 6. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Pre-Training Problem 25 and Values of Ratios.



Chi Square = 7.94^{++} (The level of significance is indicated by the number of crosses following the chi square value. One indicates $p < .05$, two indicates $p < .01$, and three indicates $p < .001$.)

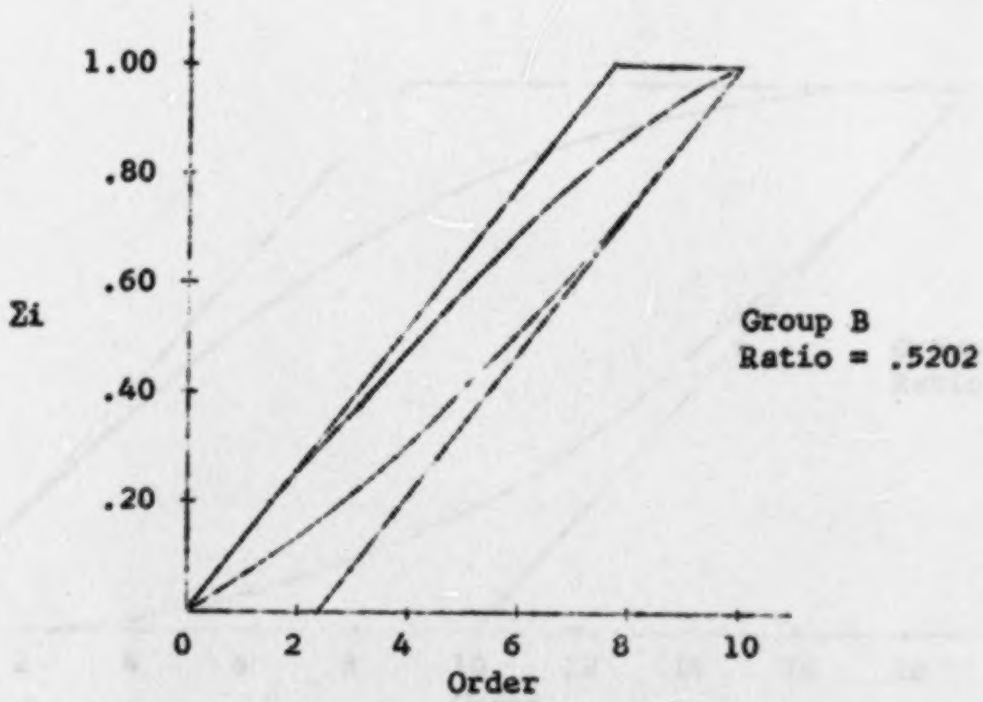
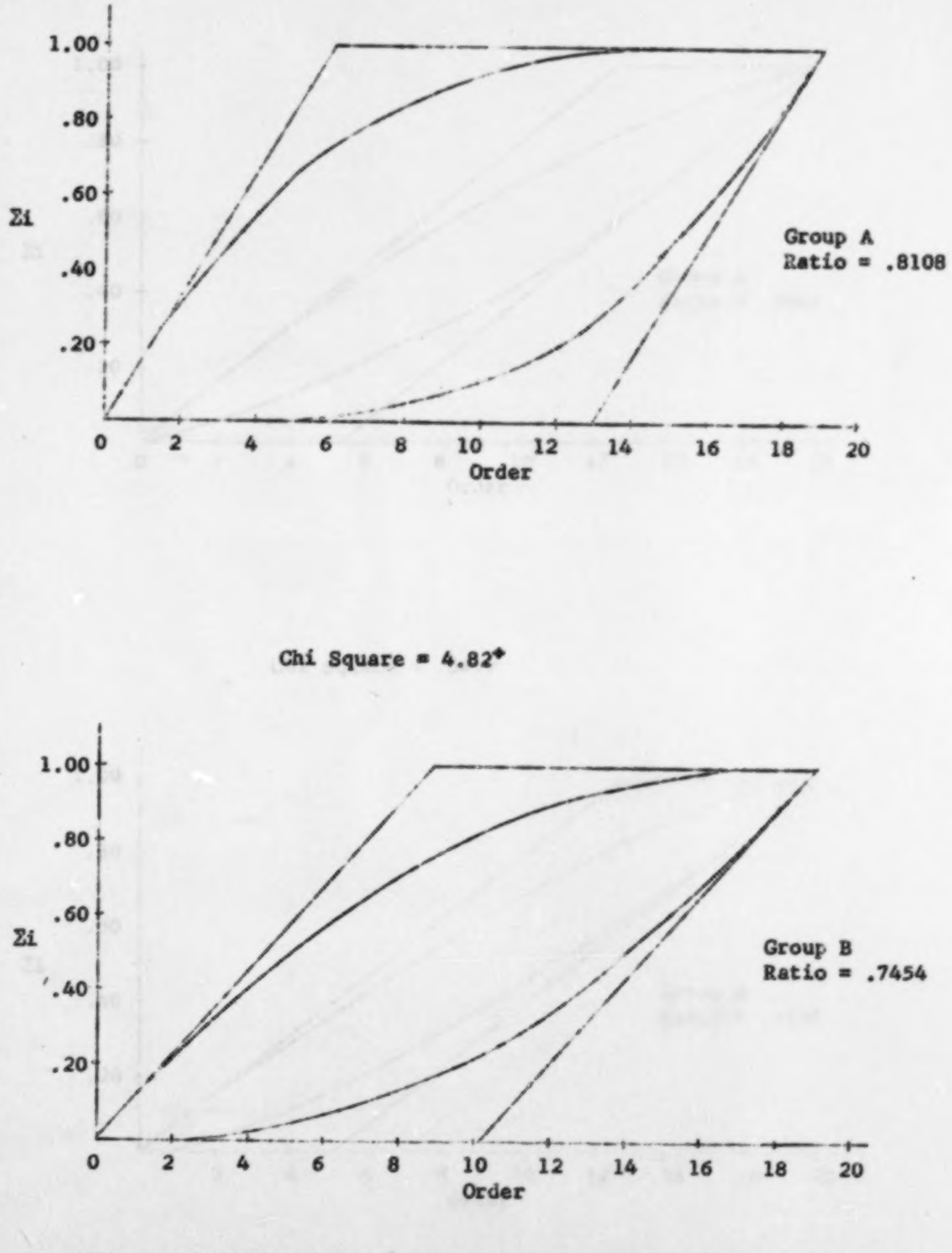


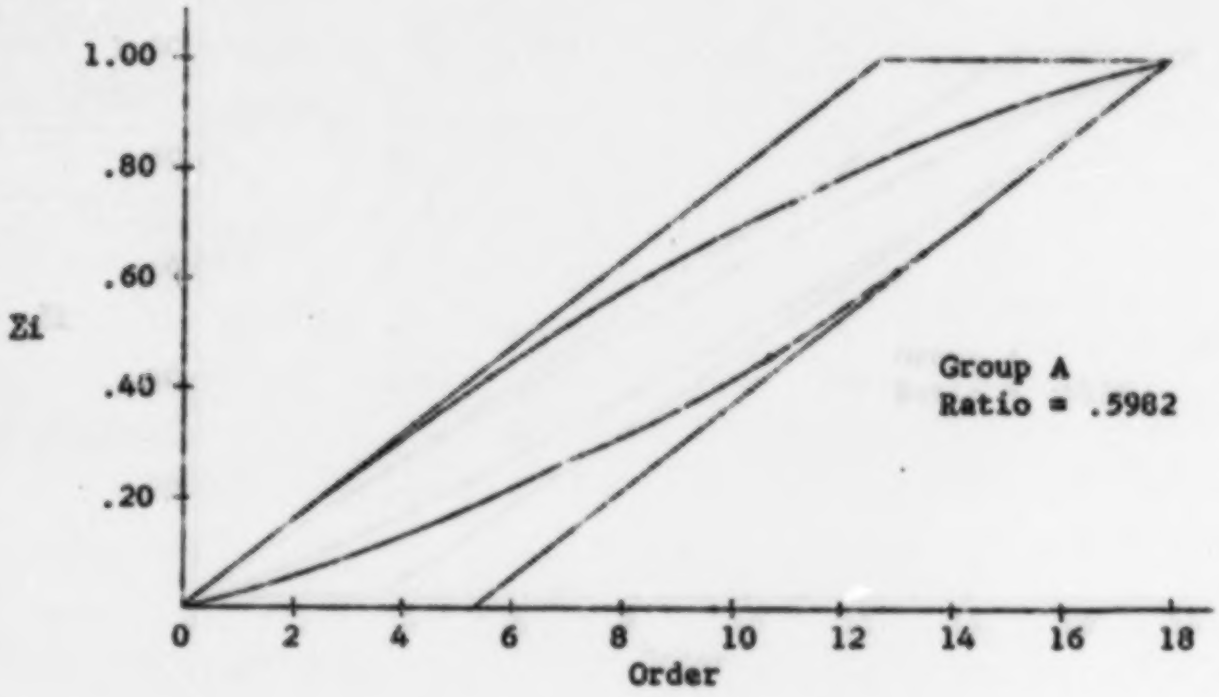
Figure 7. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Mid-Training Problem 31C' and Values of Ratios.



Chi Square = 4.82*

Figure 8. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Mid-Training Problem 35A' and Values of Ratios.

Appendix II



Chi Square = .3419

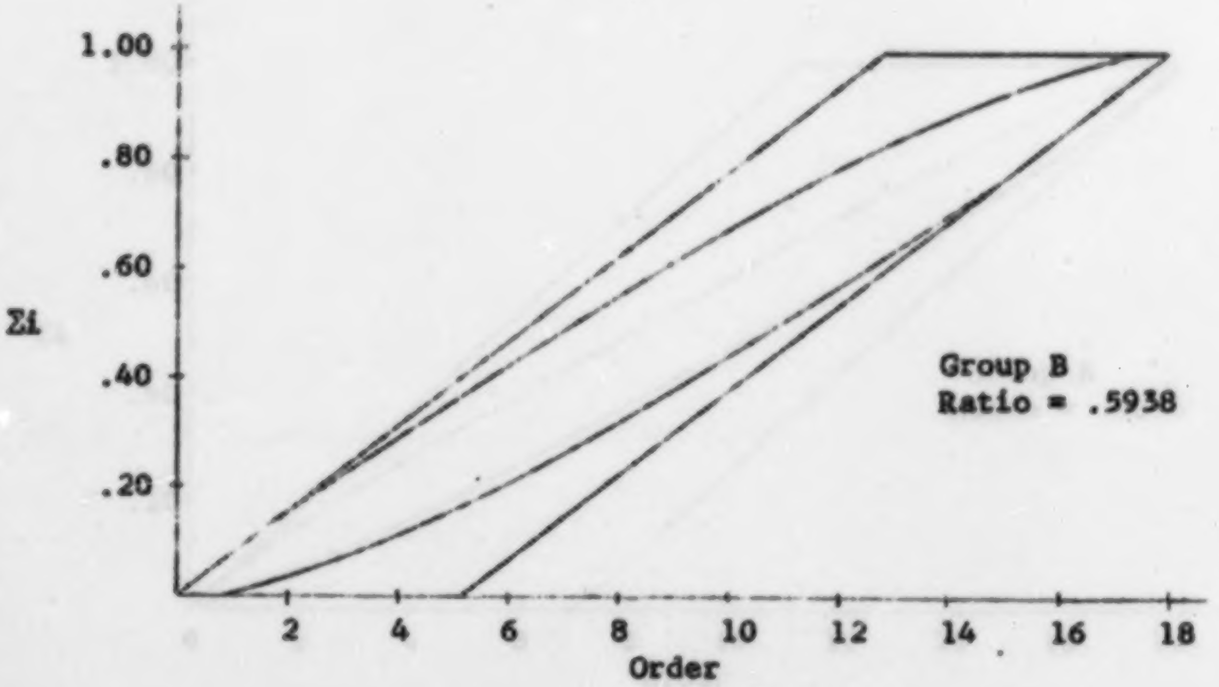
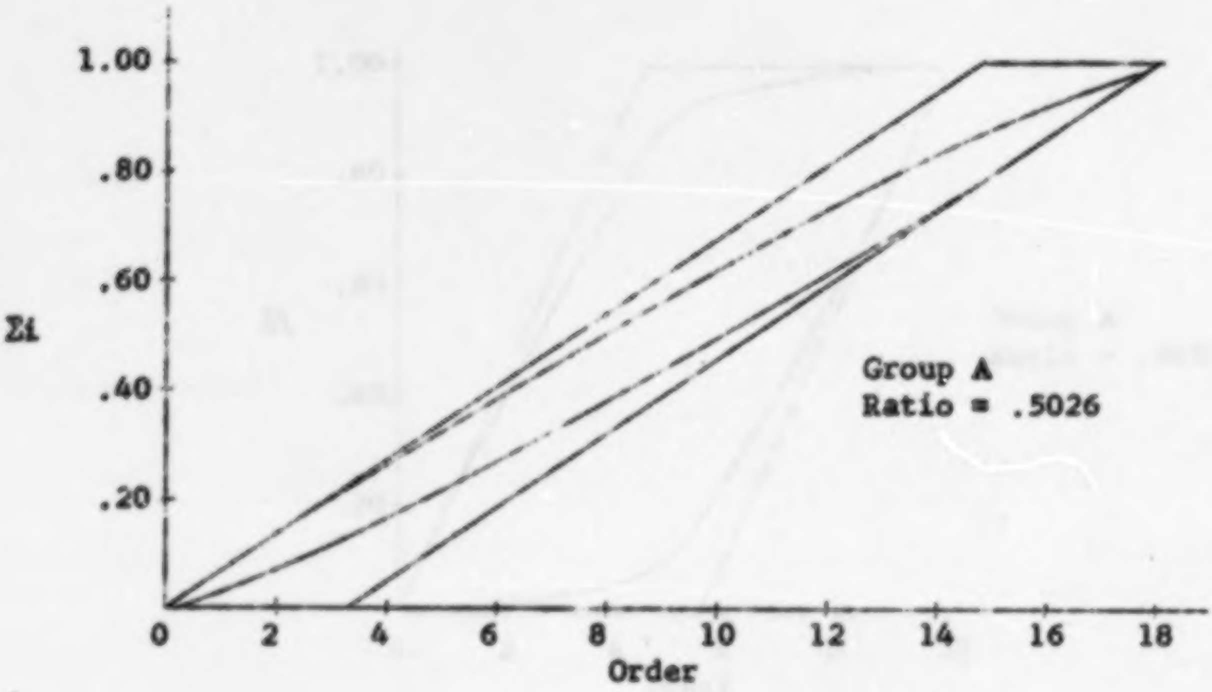


Figure 9. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Mid-Training Problem 32E and Values of Ratios.

Appendix II



Chi Square = .1114

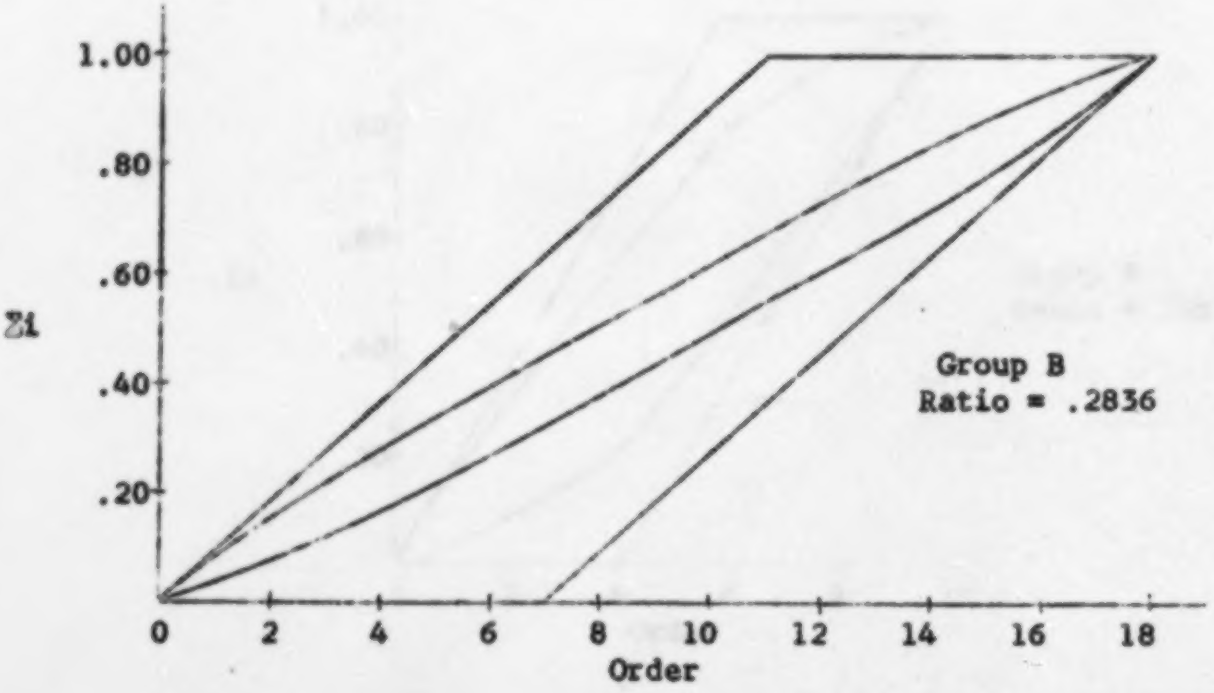
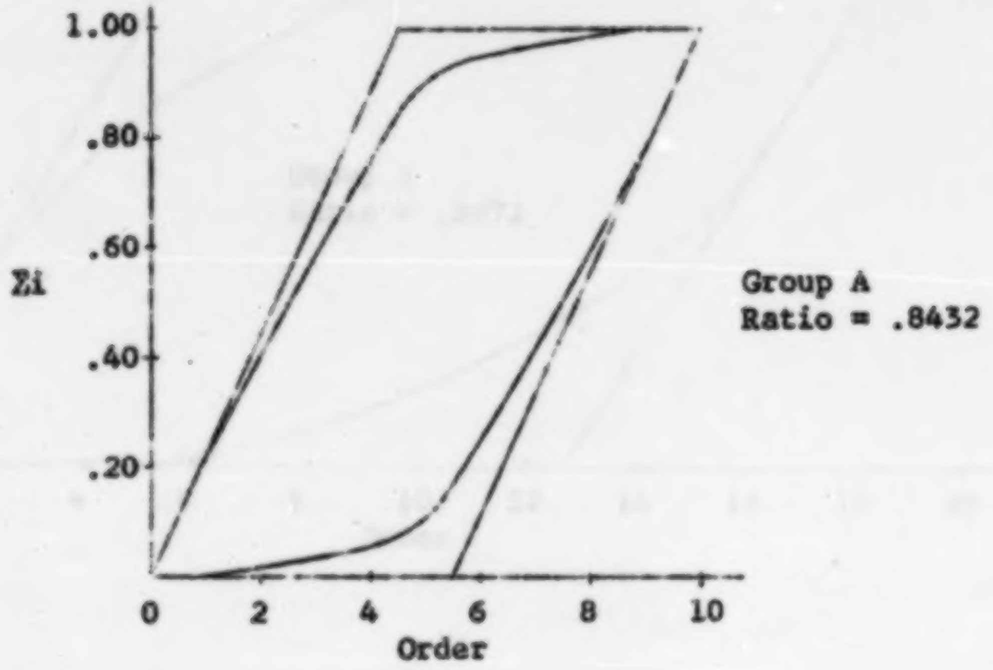


Figure 10. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Mid-Training Problem 36E and Values of Ratios.

Appendix II



Chi Square = 6.2685^{+}

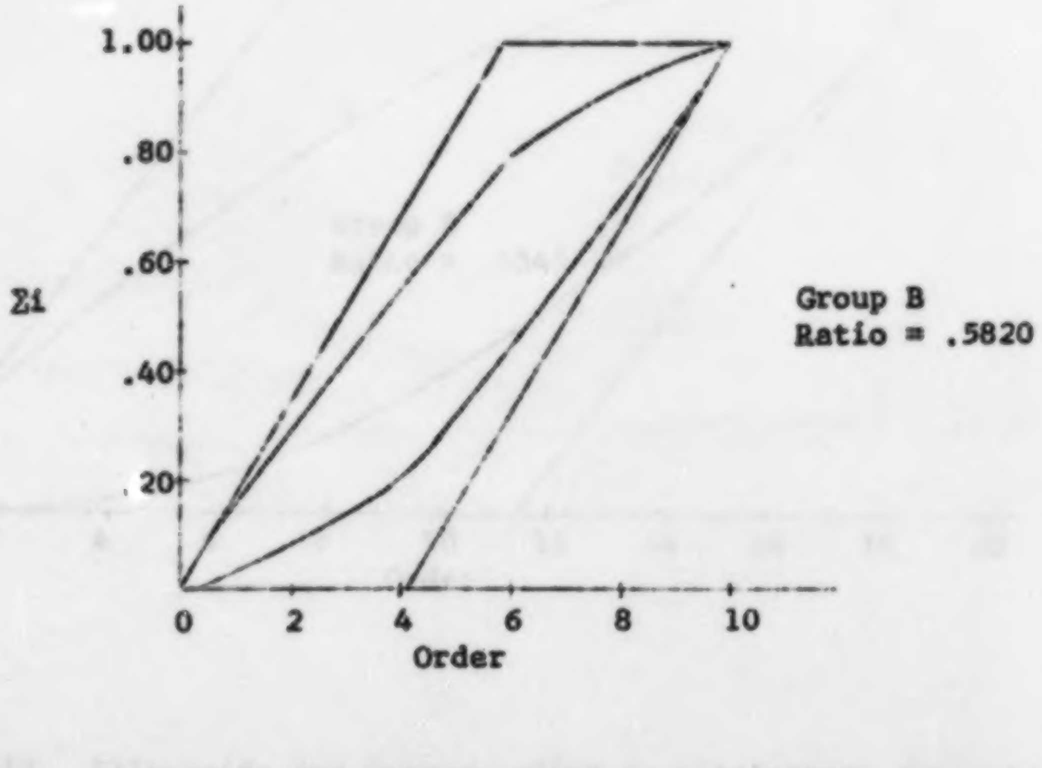
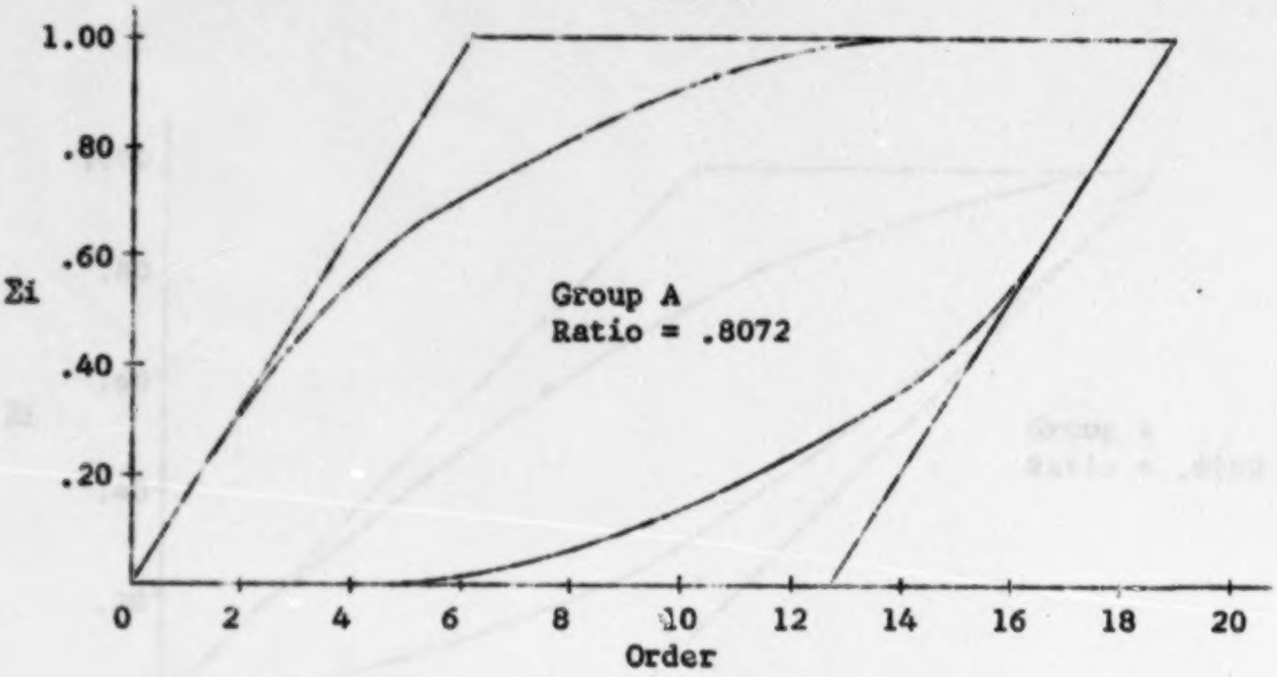


Figure 11. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Mid-Training Problem 31C' and Values of Ratios.

Appendix II



Chi Square = 4.6298^{+}

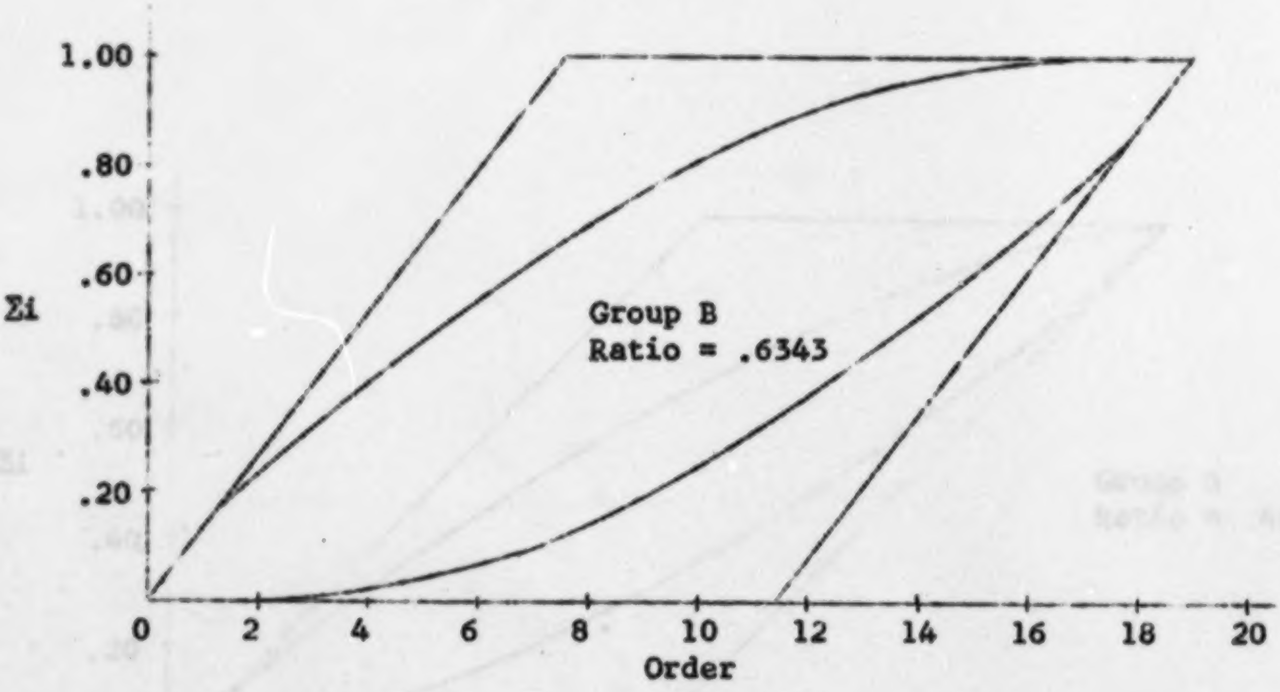
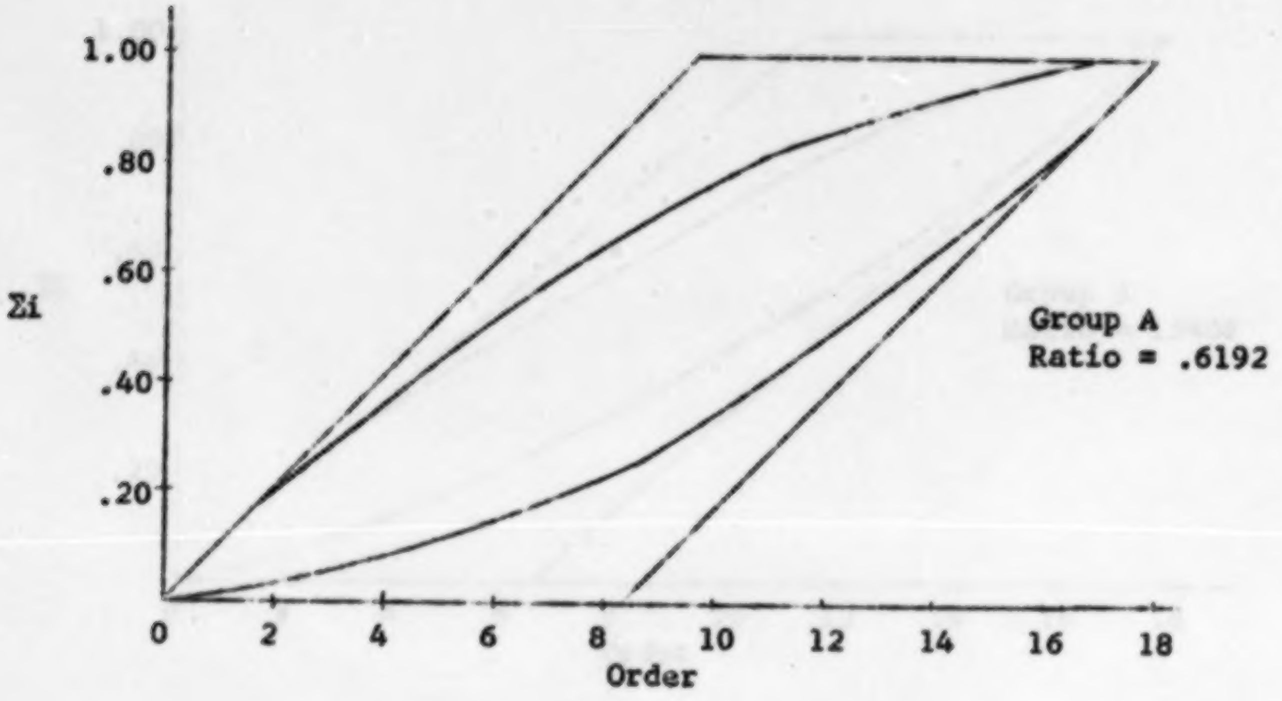


Figure 12. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Mid-Training Problem 35A' and Values of Ratios.

Appendix II



Chi Square = .9749

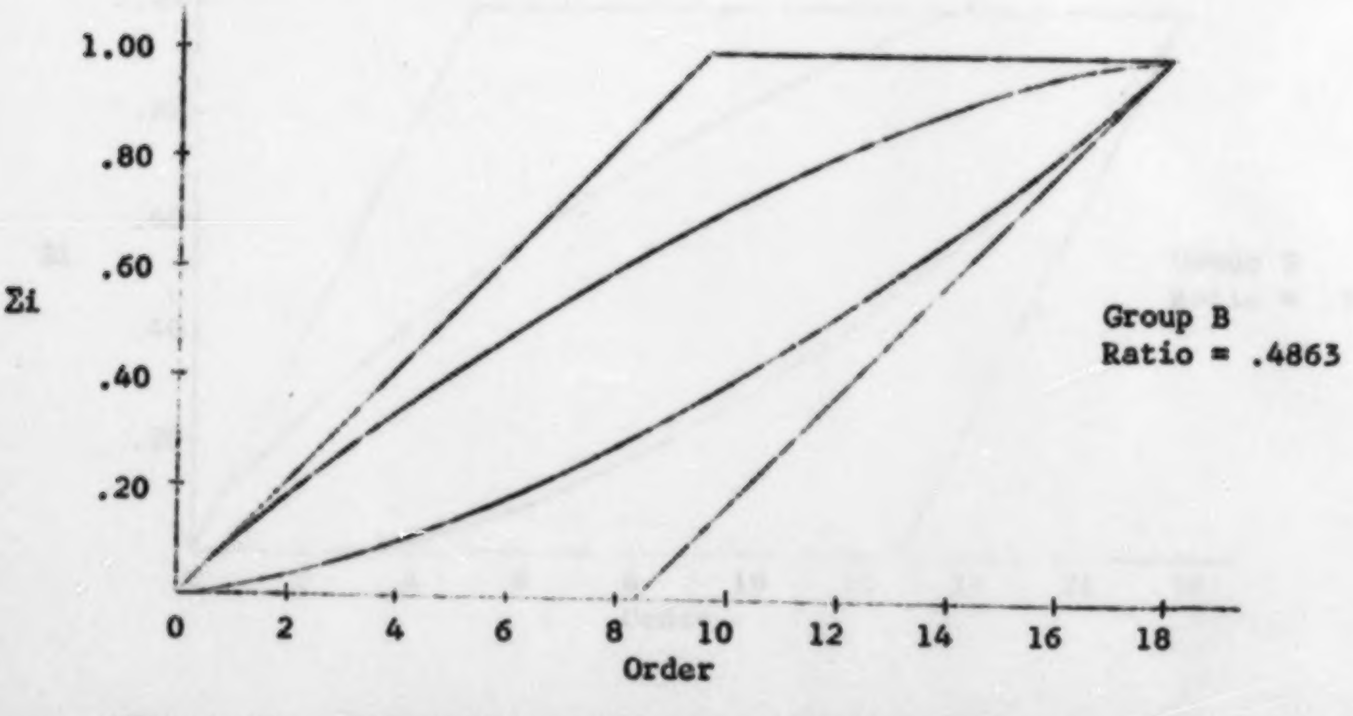
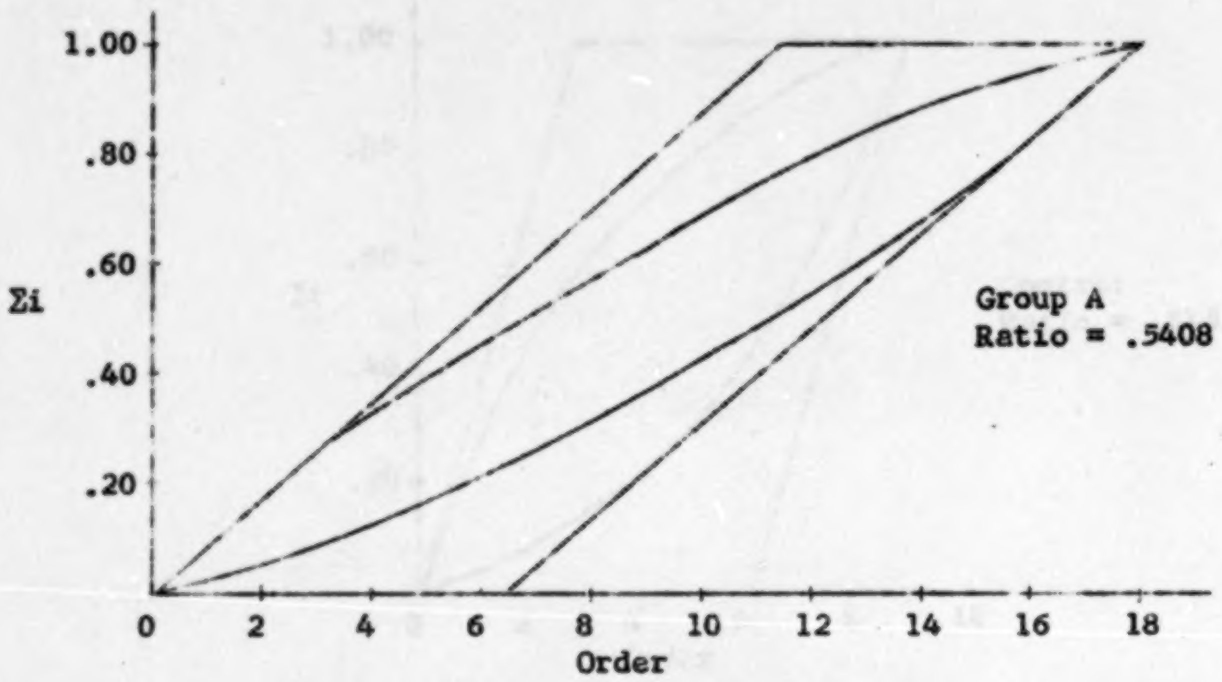


Figure 13. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Mid-Training Problem 32E and Values of Ratios.



Chi Square = 3.2514

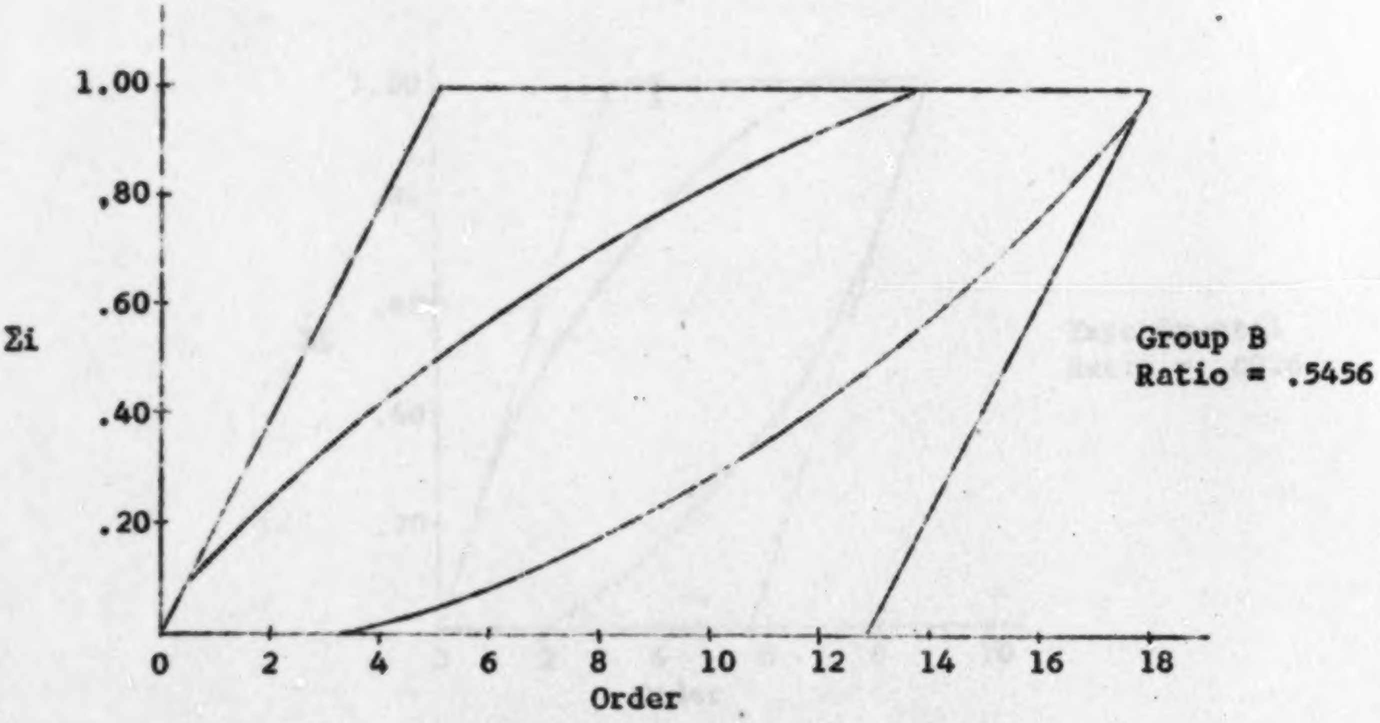
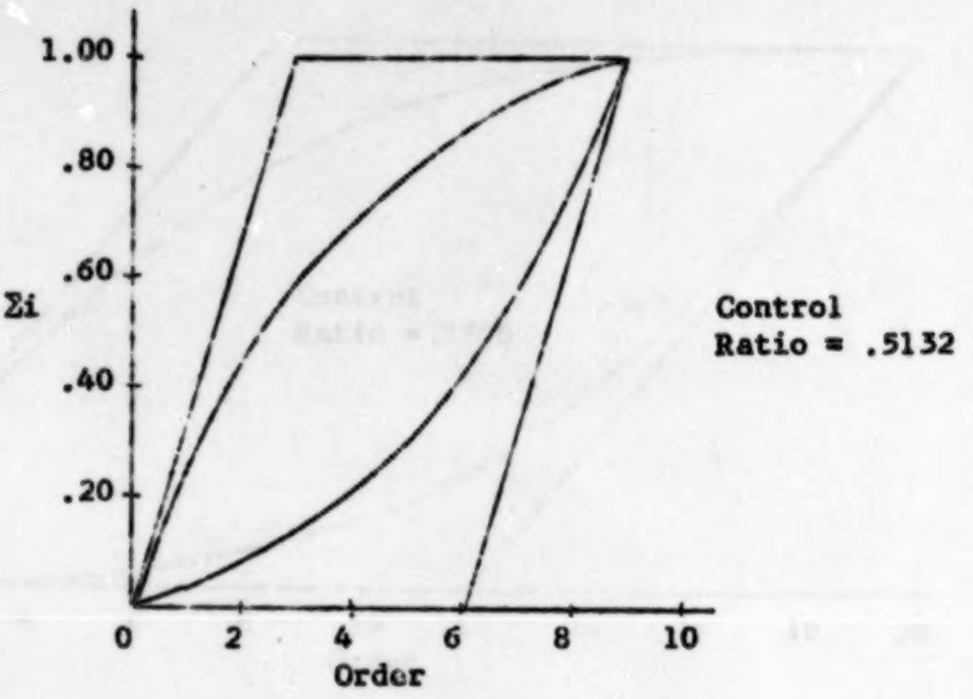


Figure 14. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Mid-Training Problem 36E and Values of Ratios.

Appendix II



Chi Square = .8093

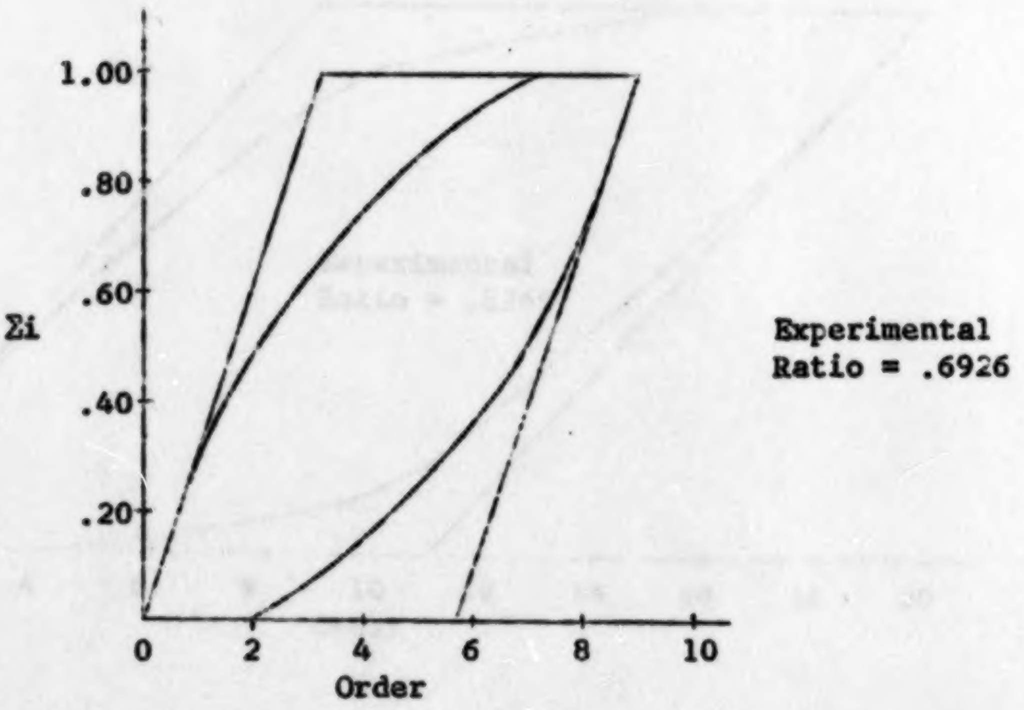
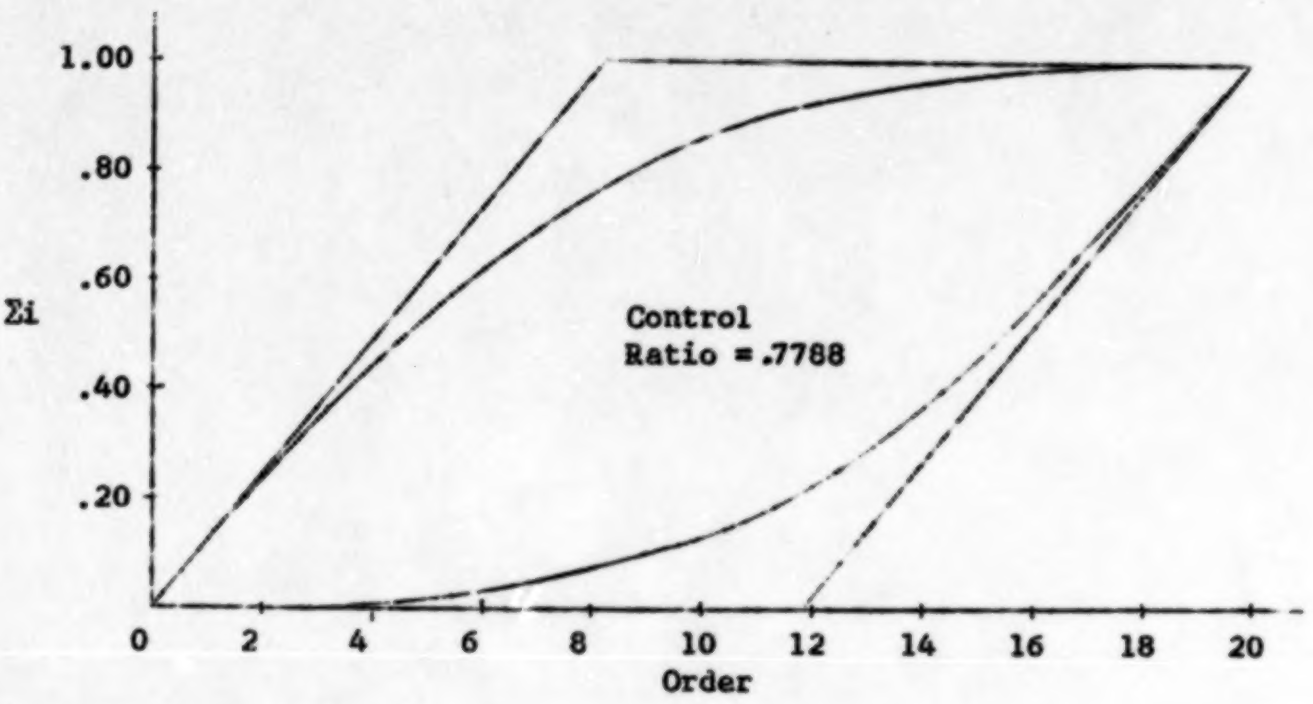


Figure 15. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 1 and Values of Ratios.

Appendix II



Chi Square = .2041

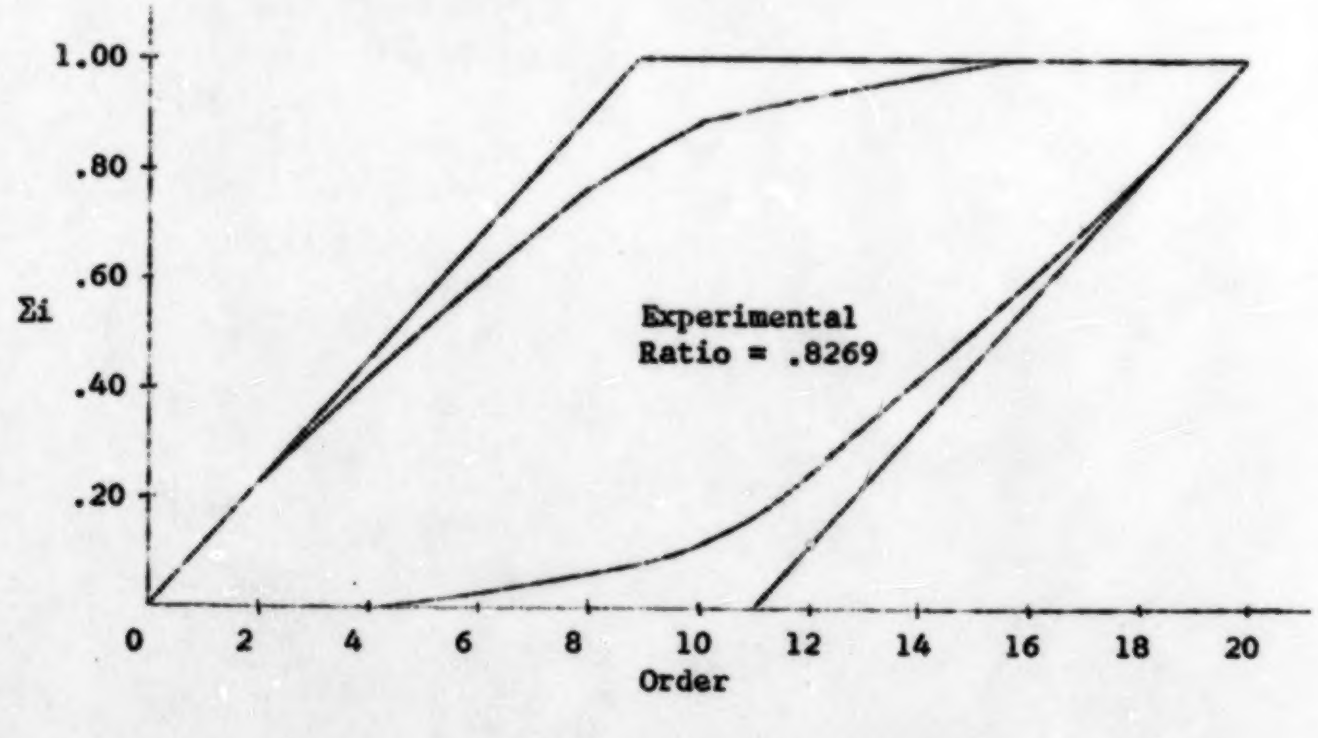


Figure 16. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 19 and Values of Ratios.

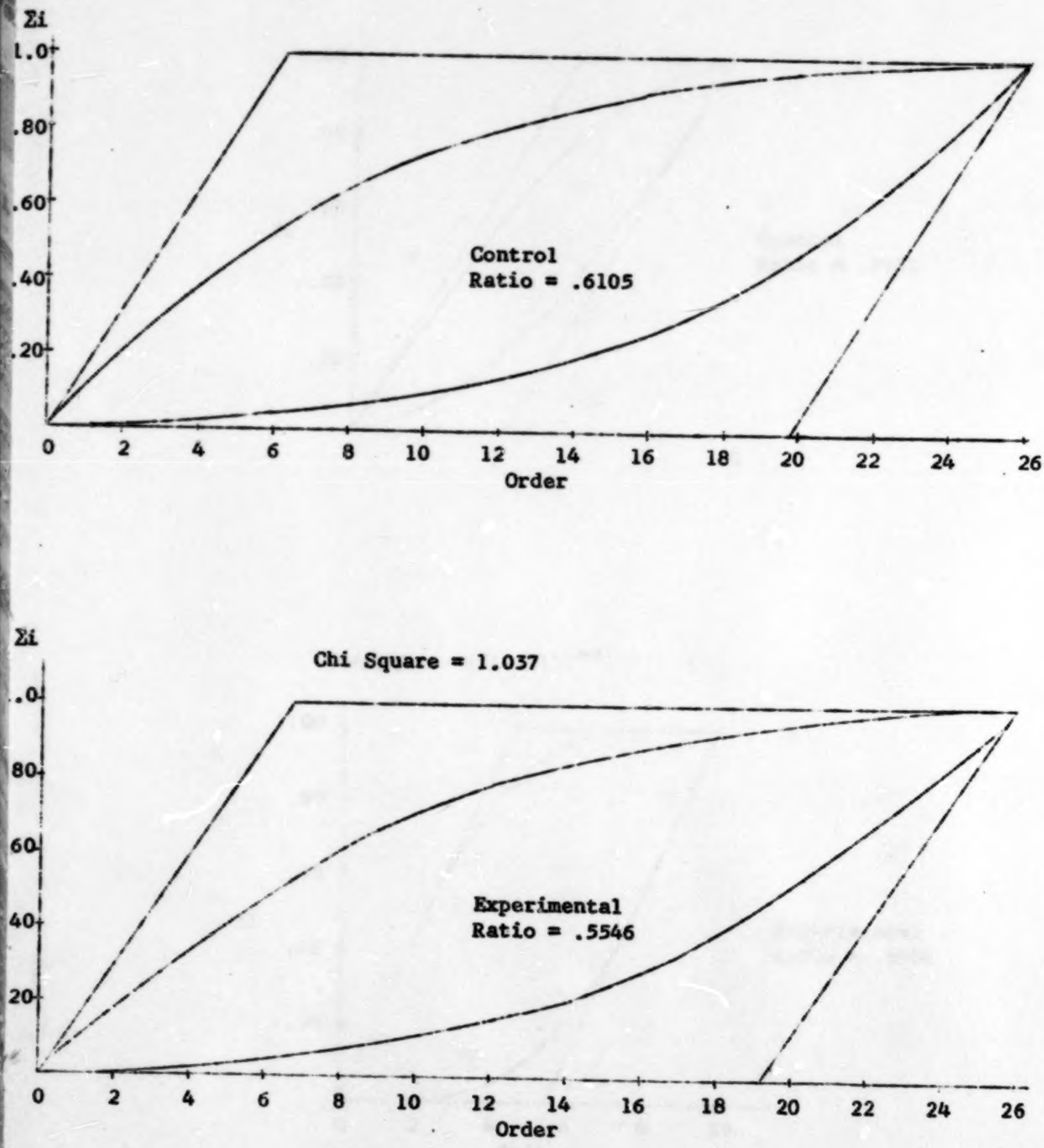
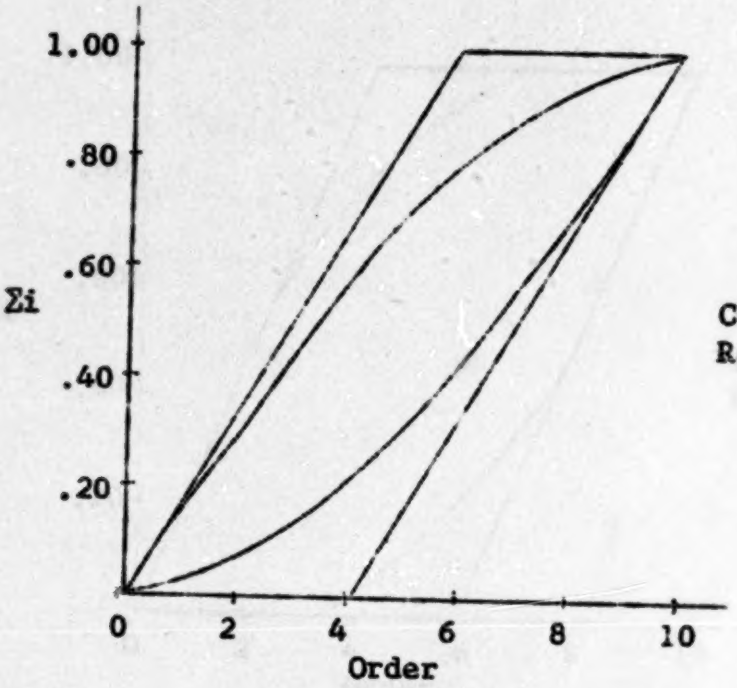
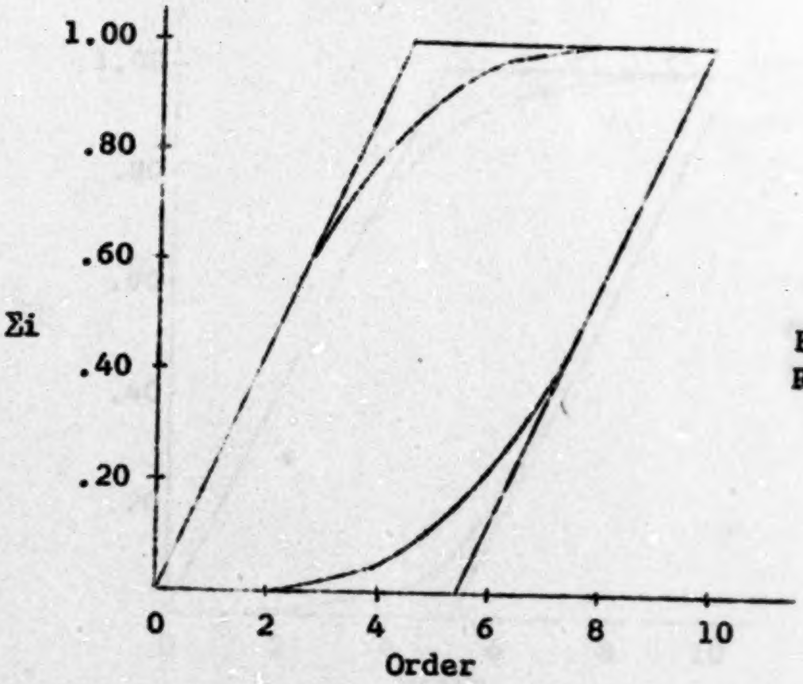


Figure 17. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 25 and Values of Ratios.



Control
Ratio = .5833

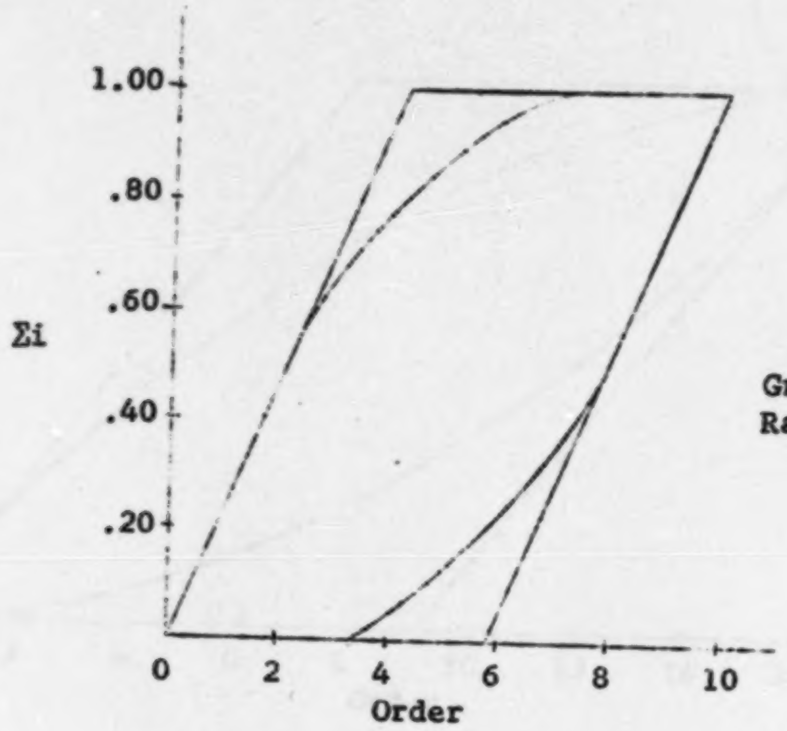
Chi Square = 10.9172^{++}



Experimental
Ratio = .8904

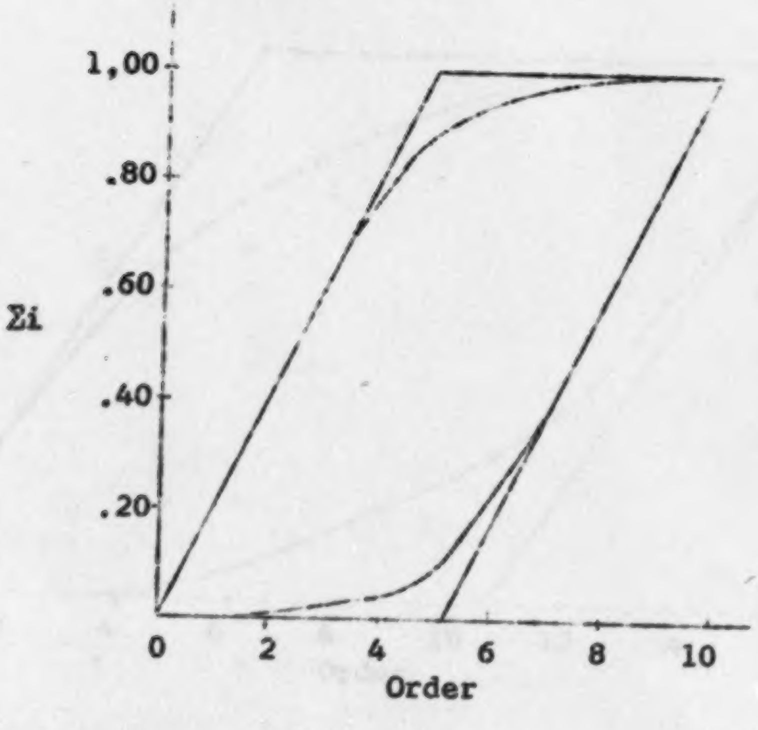
Figure 18. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 31D' and Values of Ratios.

Appendix II



Group A
Ratio = .8856

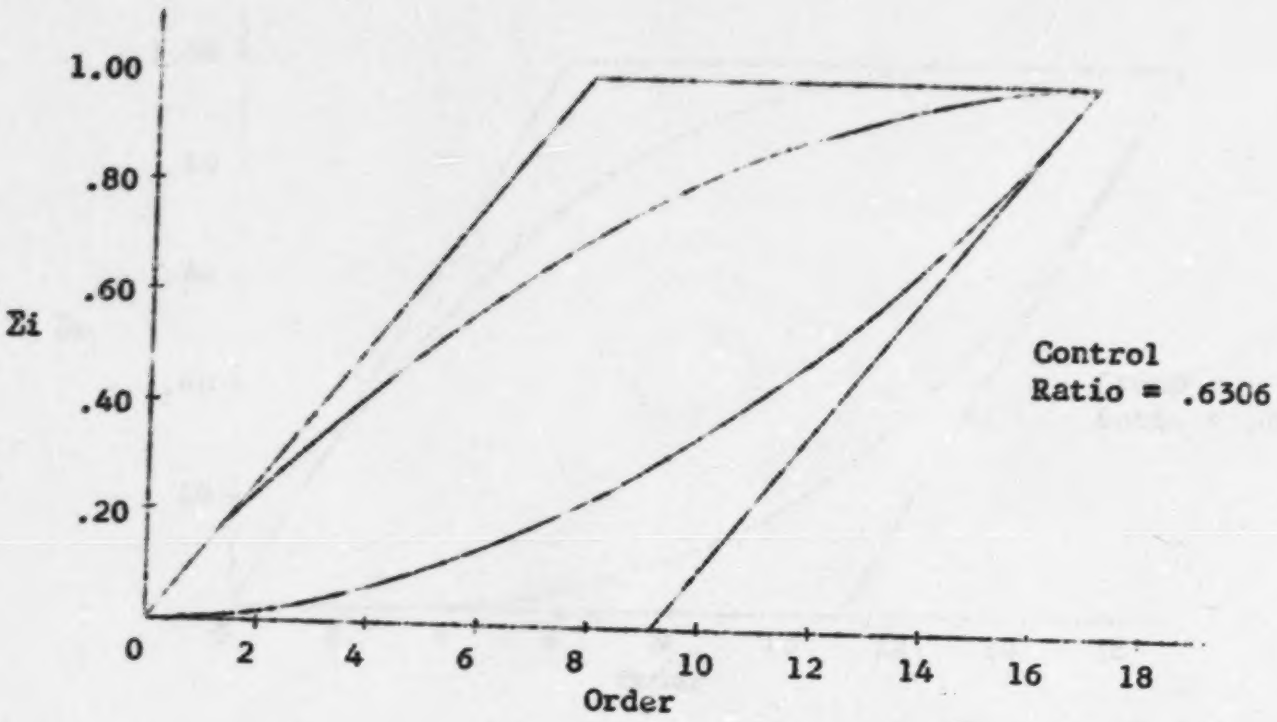
Chi Square = .5673



Group B
Ratio = .9144

Figure 19. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Post-Training Problem 31D' and Values of Ratios.

Appendix II



Chi Square = 7.7968^{++}

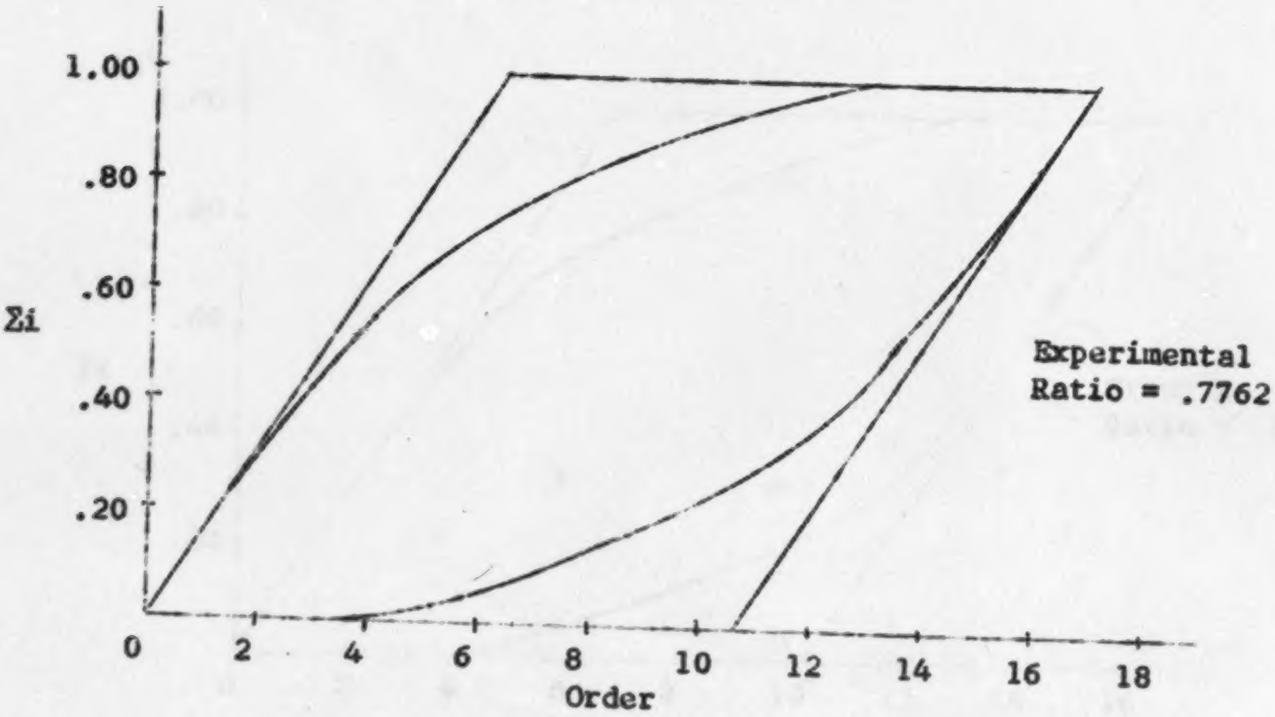
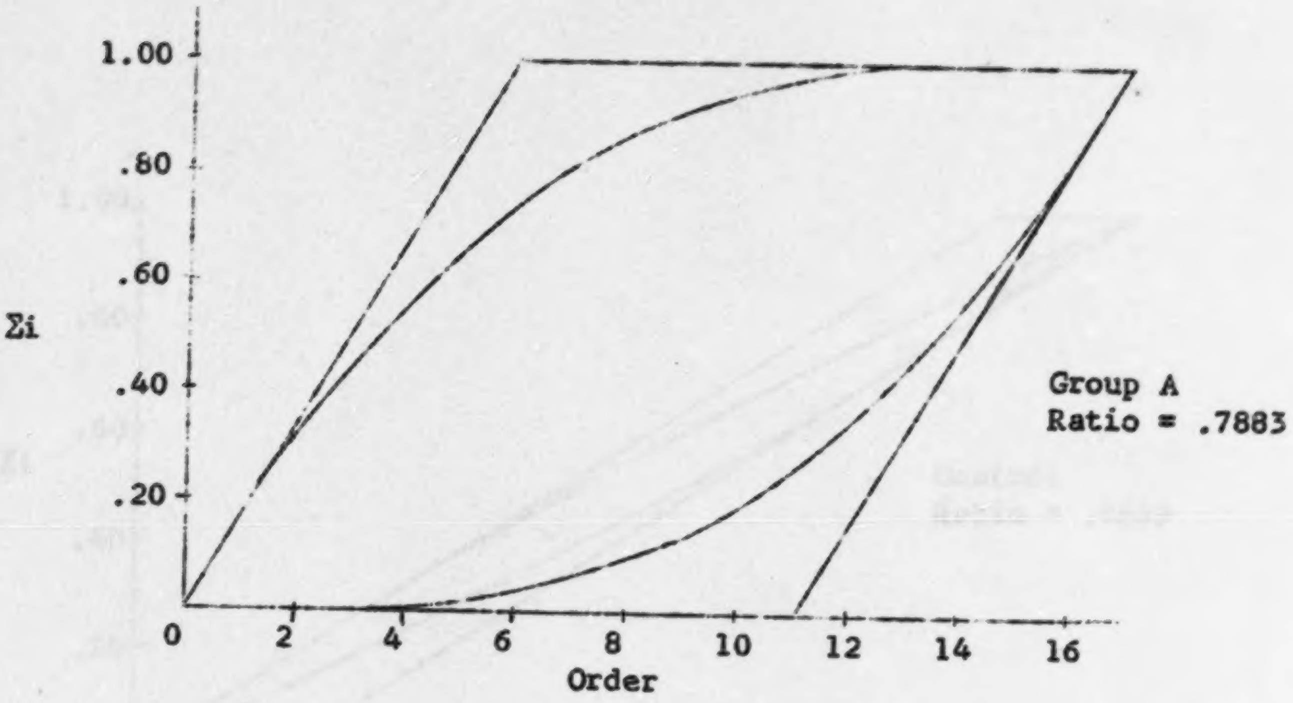


Figure 20. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 35B' and Values of Ratios.

Appendix II



Chi Square = .3190

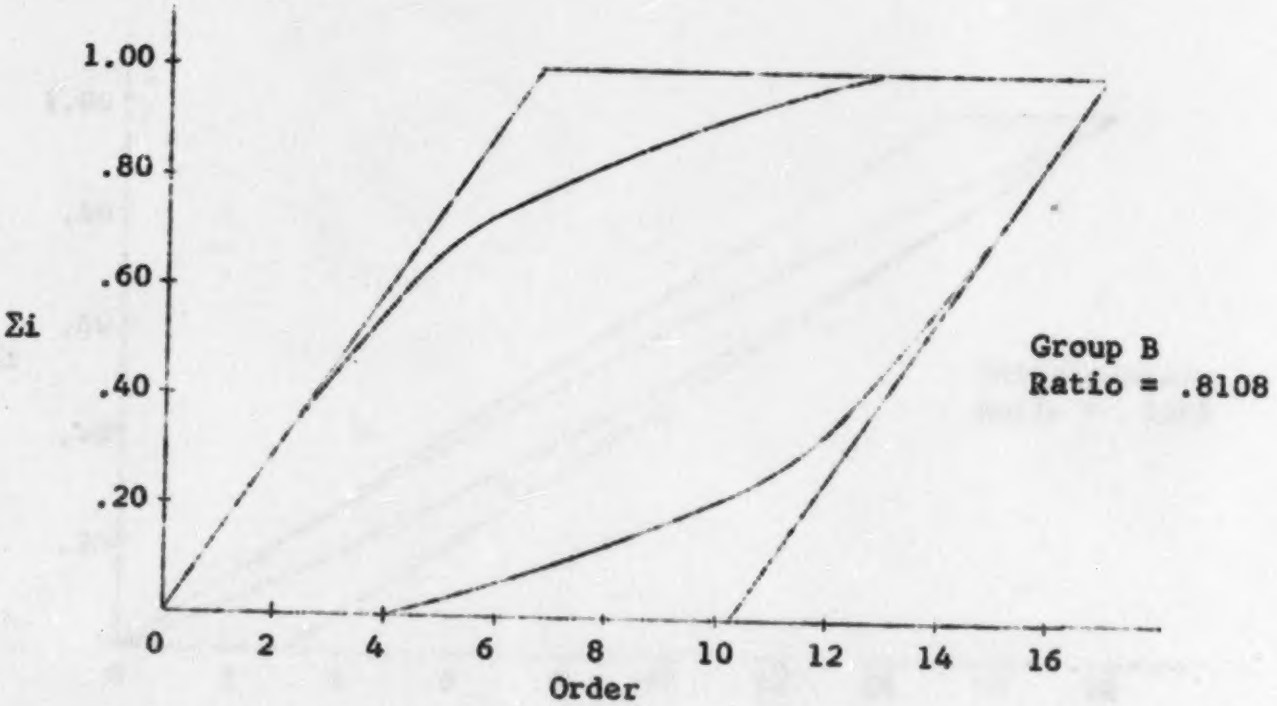
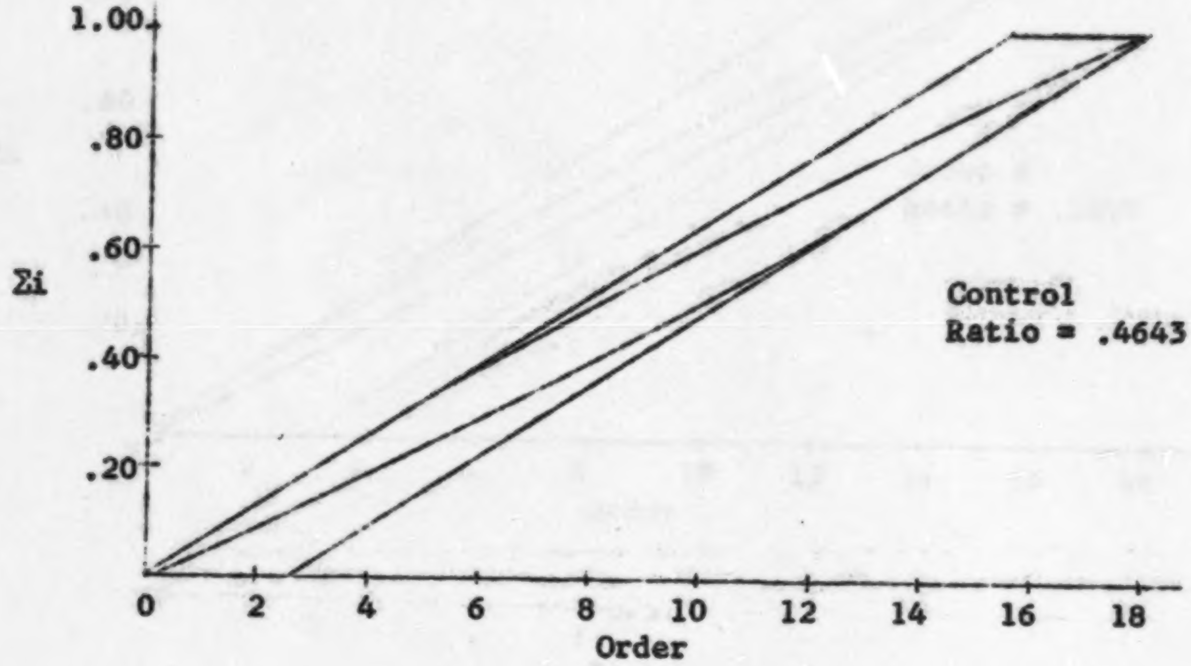


Figure 21. Ellipsoids and Corresponding Parallelograms Generated by Performance of Ignatius Experimental Groups A and B on Post-Training Problem 35B' and Values of Ratios.

Appendix II



Chi Square = .0121

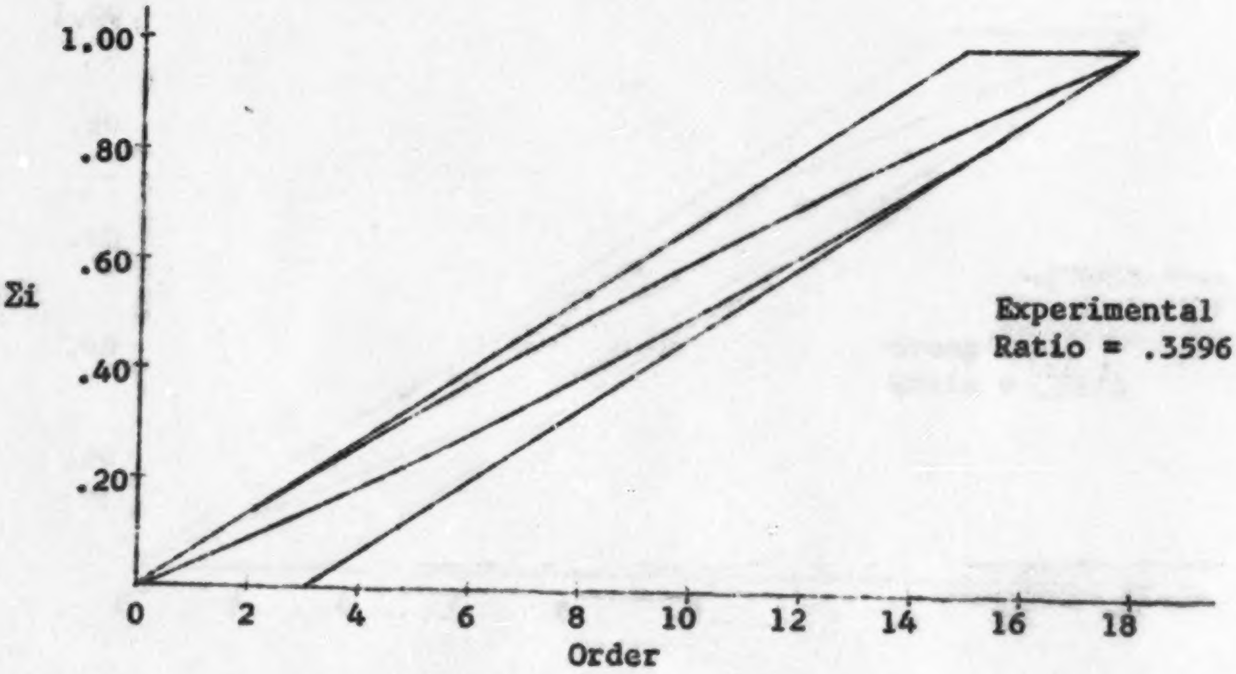
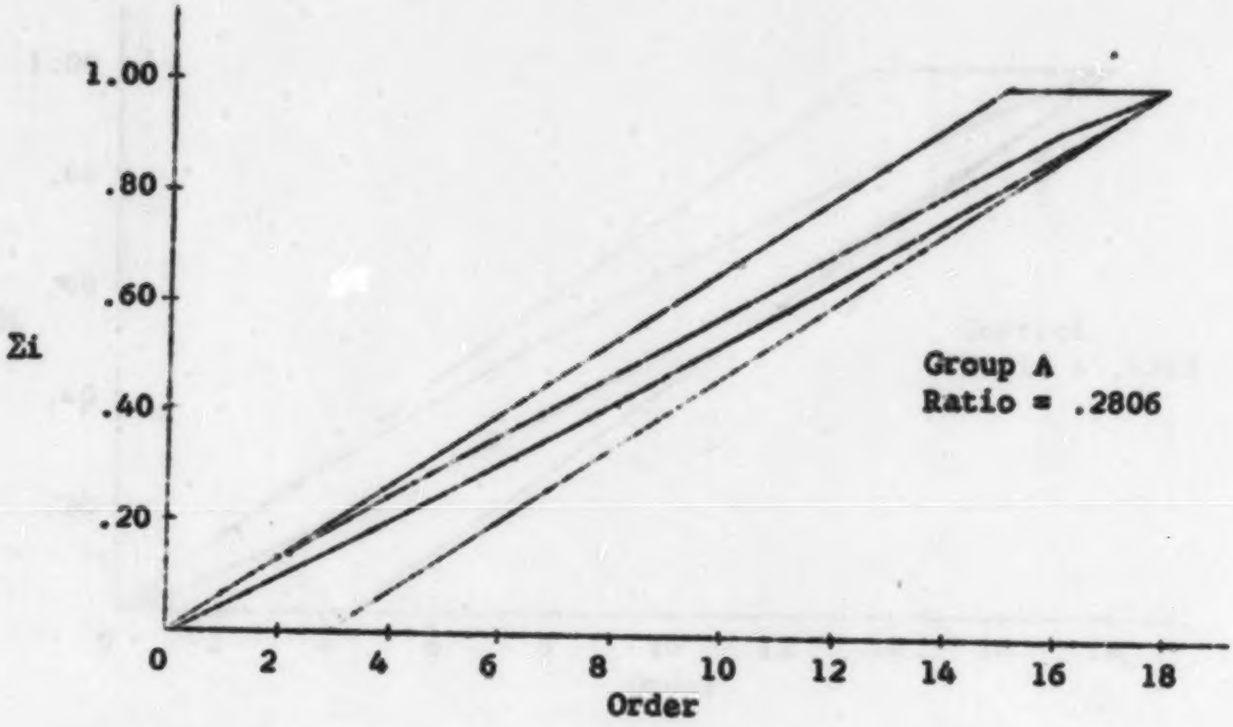


Figure 22. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 32F and Values of Ratios.



Chi Square = 1.2413

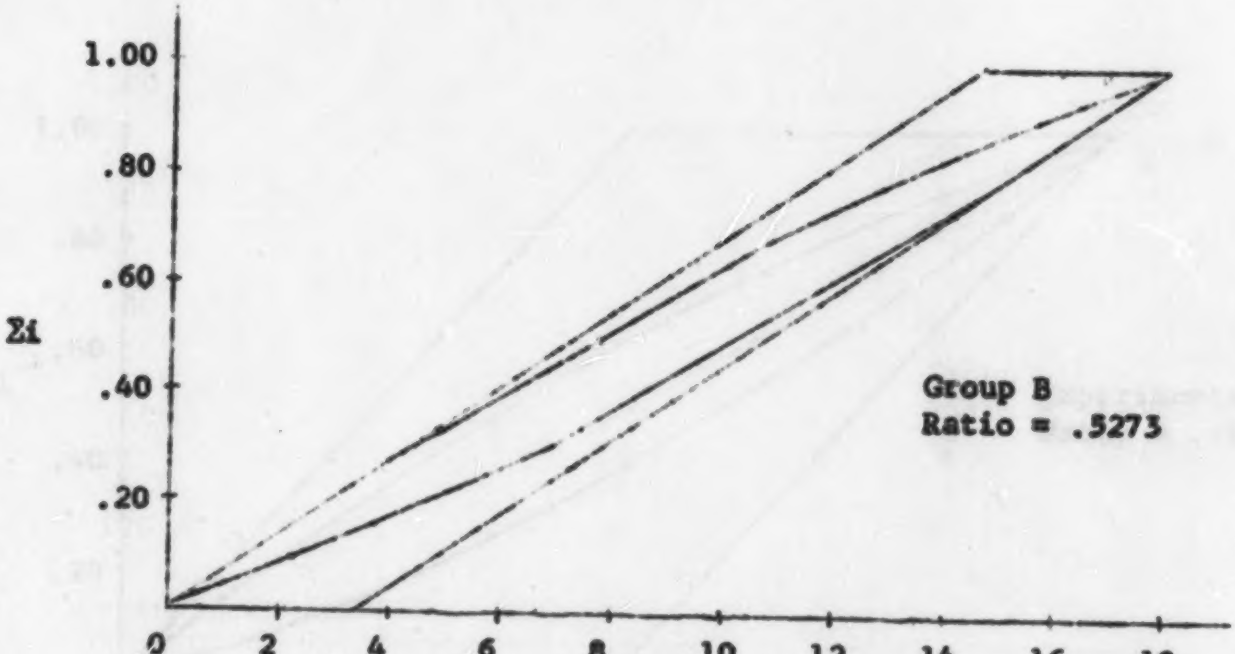
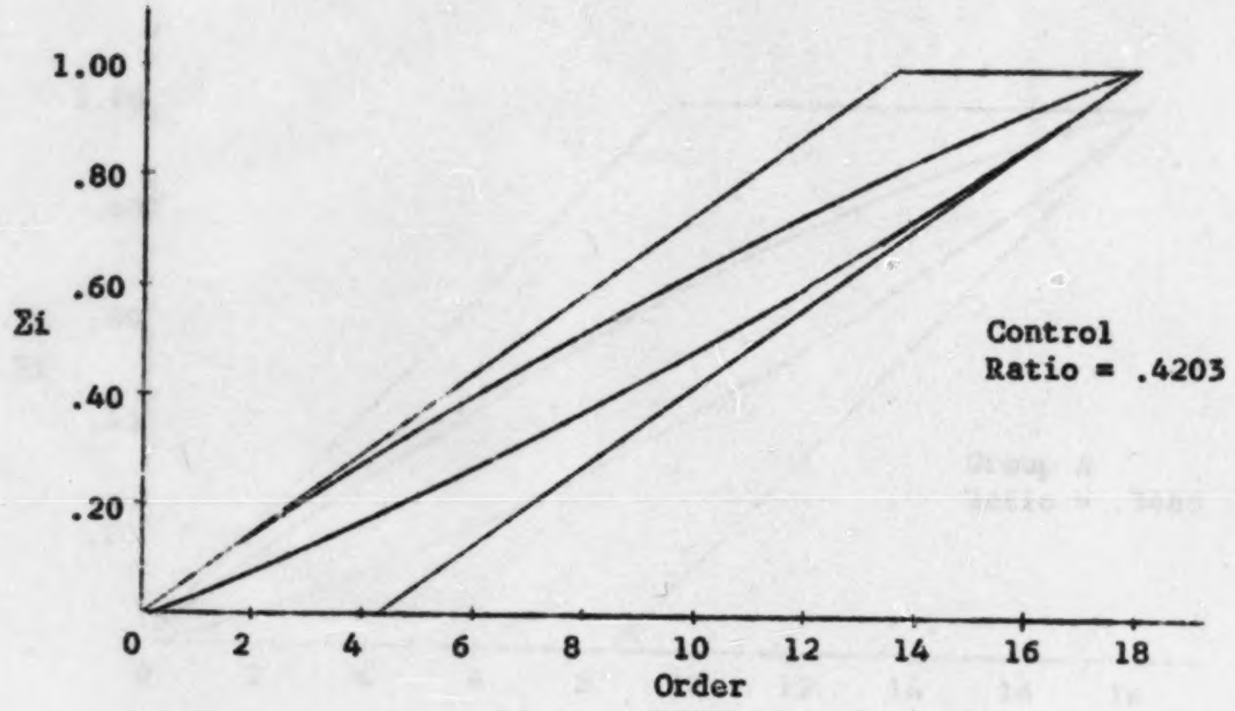


Figure 23. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Post-Training Problem 32F and Values of Ratios.

Appendix II



Chi Square = .5254

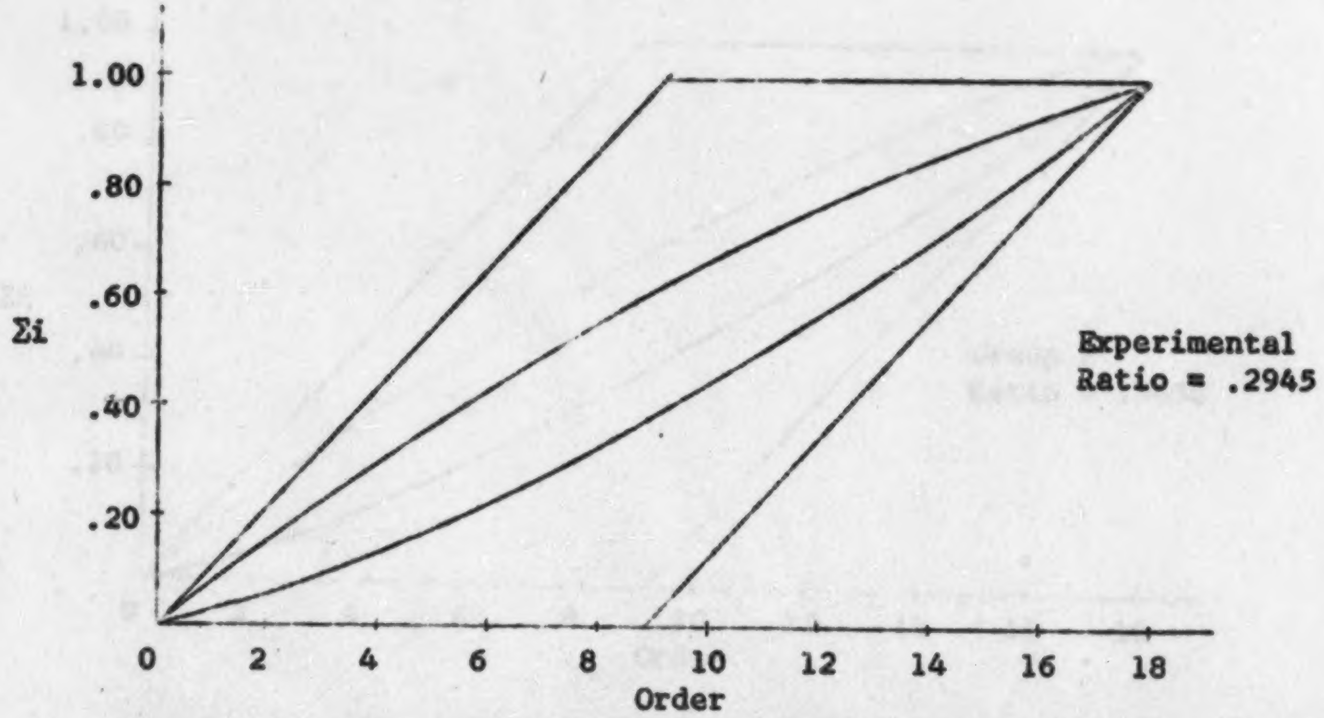
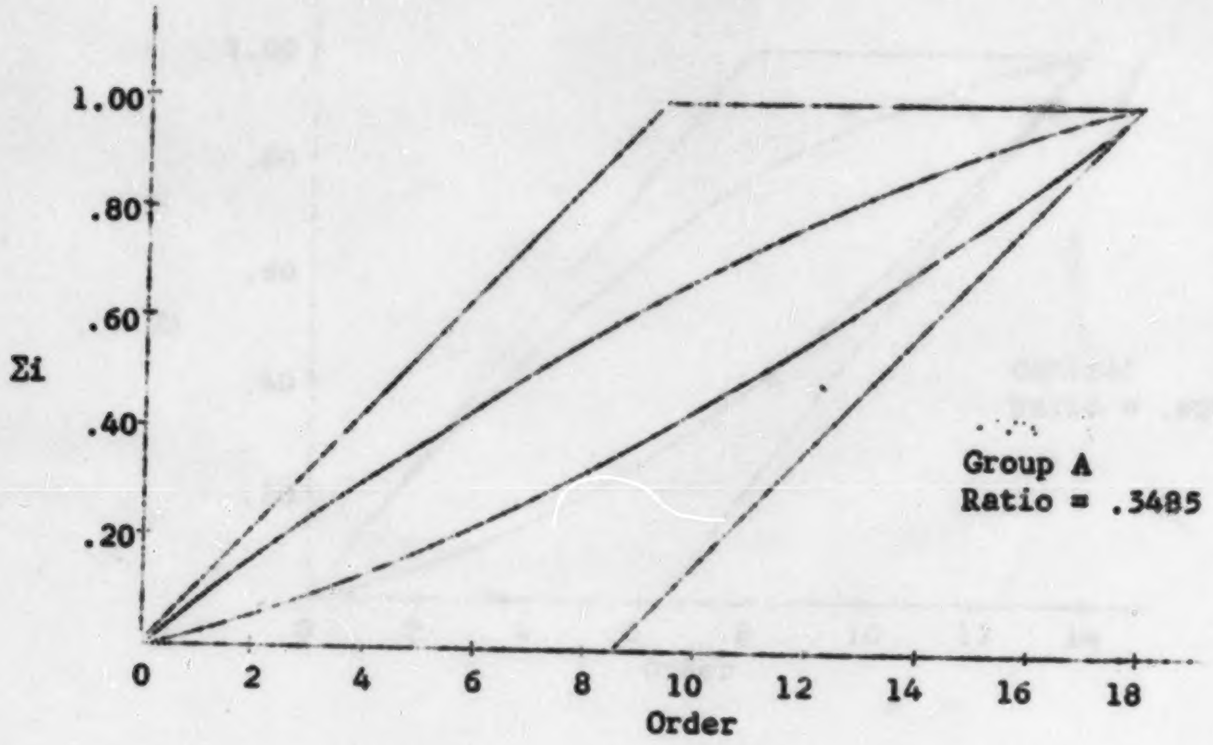


Figure 24. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 36F and Values of Ratios.

Appendix II



Chi Square = .0380

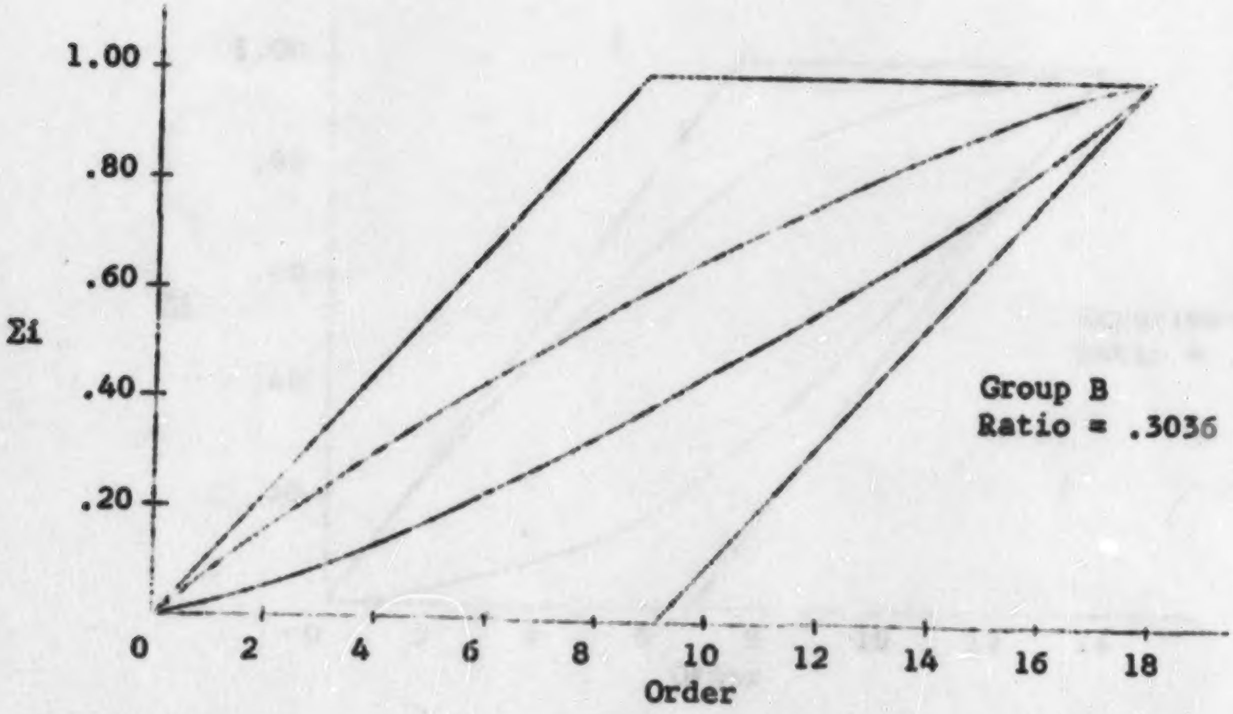
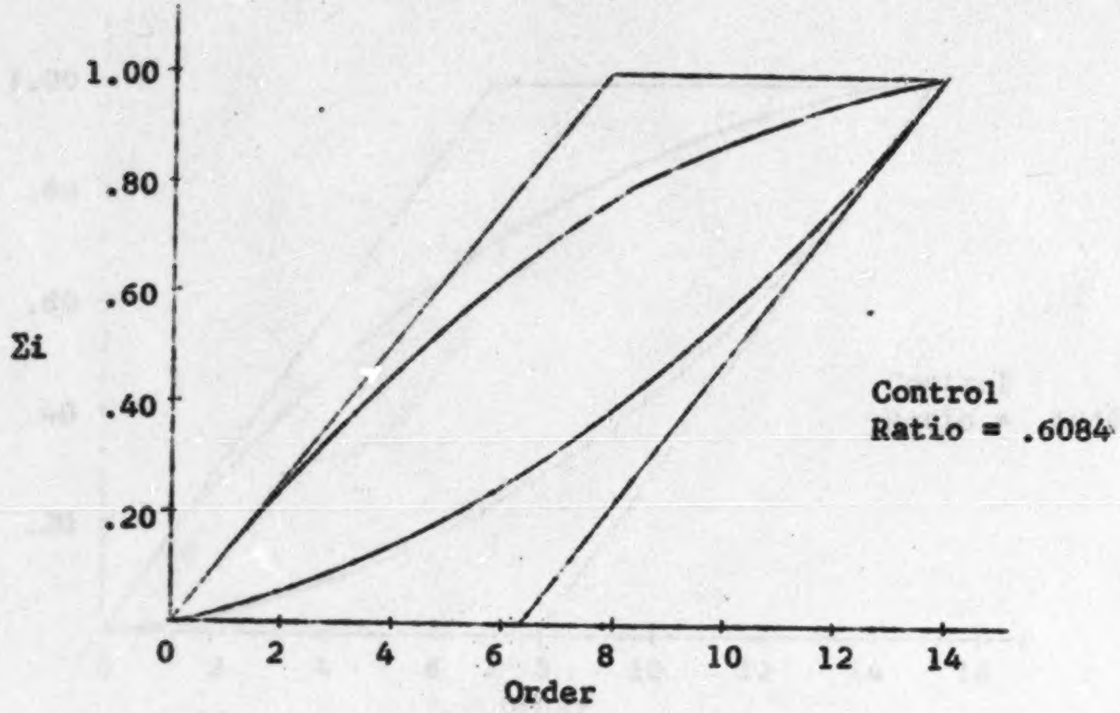


Figure 25. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Experimental Groups A and B on Post-Training Problem 36F and Values of Ratios.

Appendix II



Chi Square = 1.3098

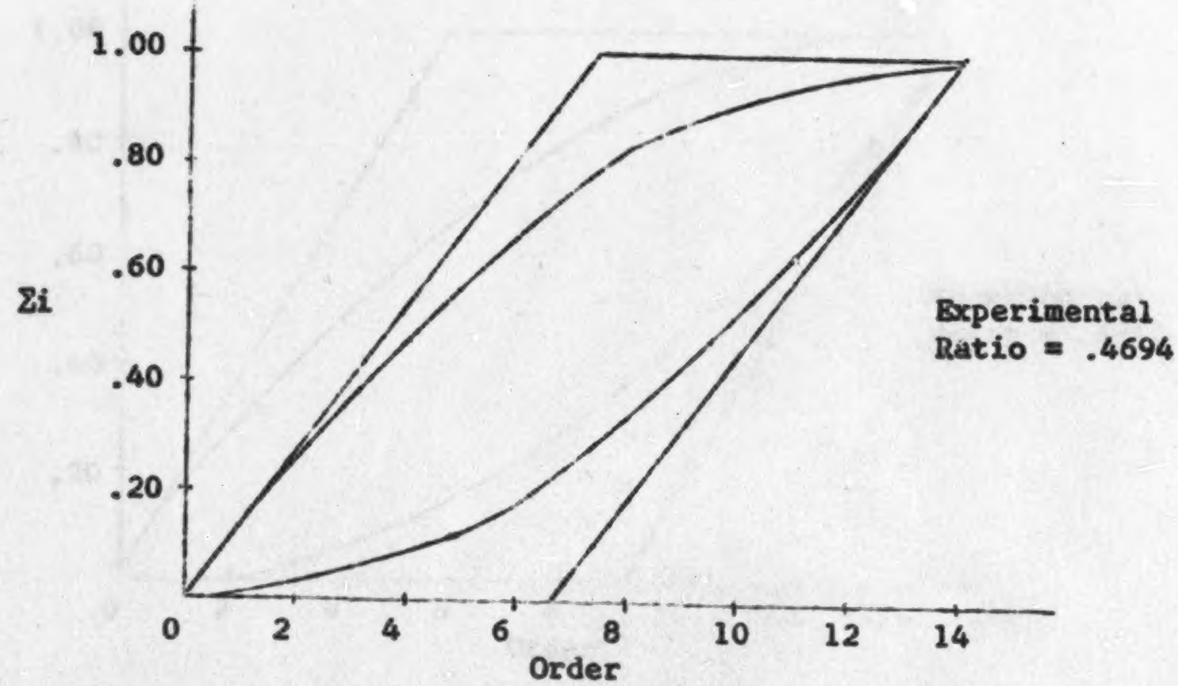
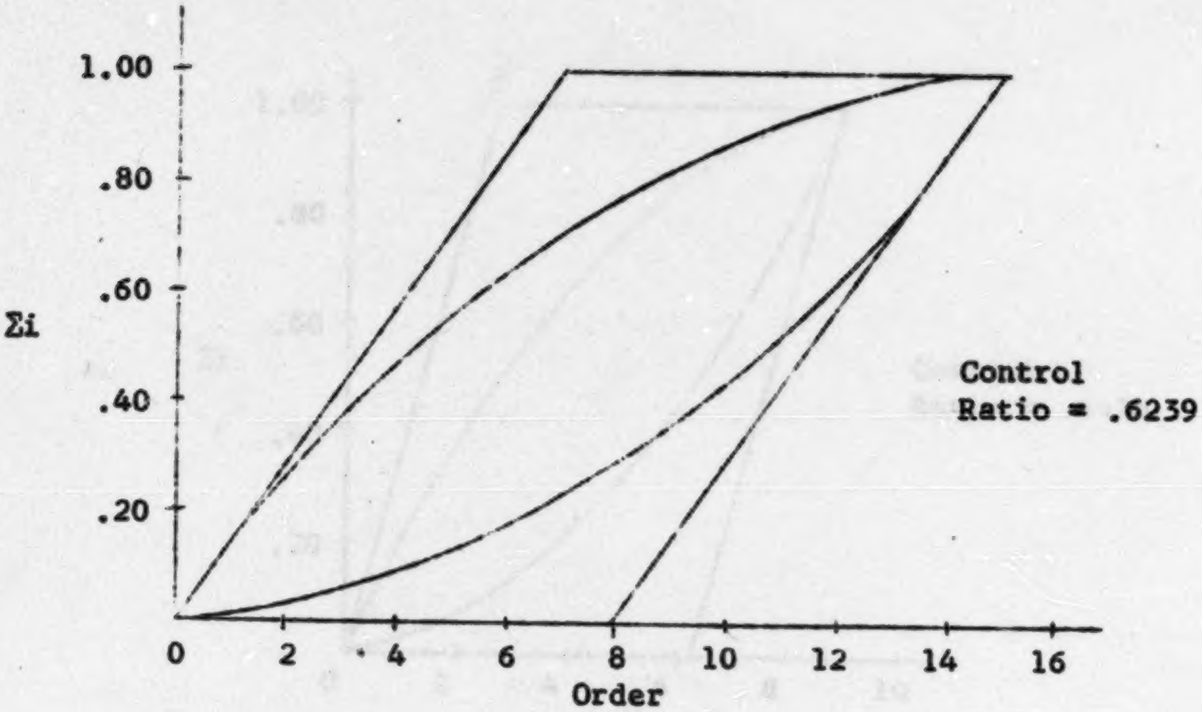


Figure 26. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 26 and Values of Ratios.

Appendix II



Chi Square = .7327

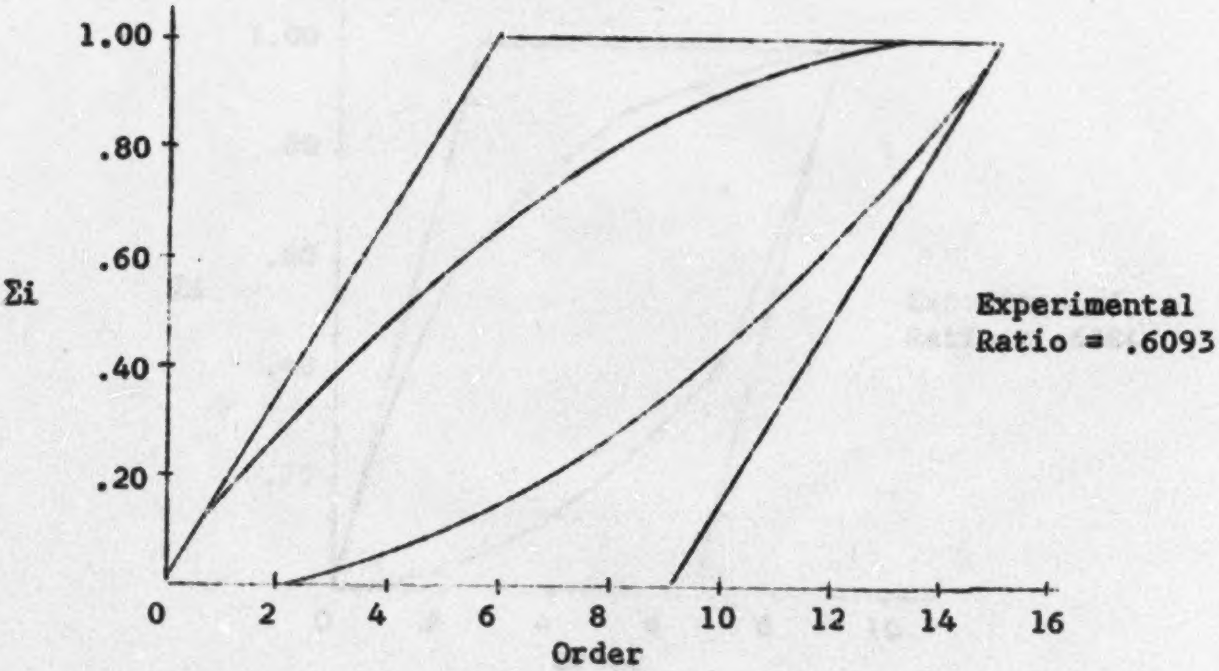
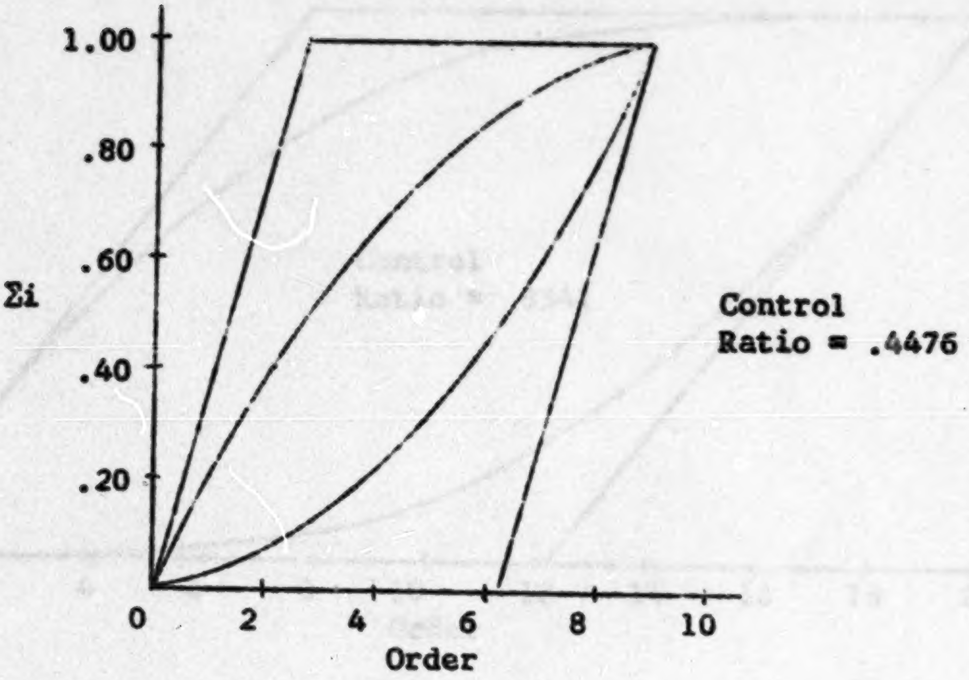


Figure 27. Ellipsoids and Corresponding Parallelograms Generated by Performances of Ignatius Control and Experimental Subjects on Post-Training Problem 41A and Values of Ratios.

Appendix II



Chi Square = 2.1437

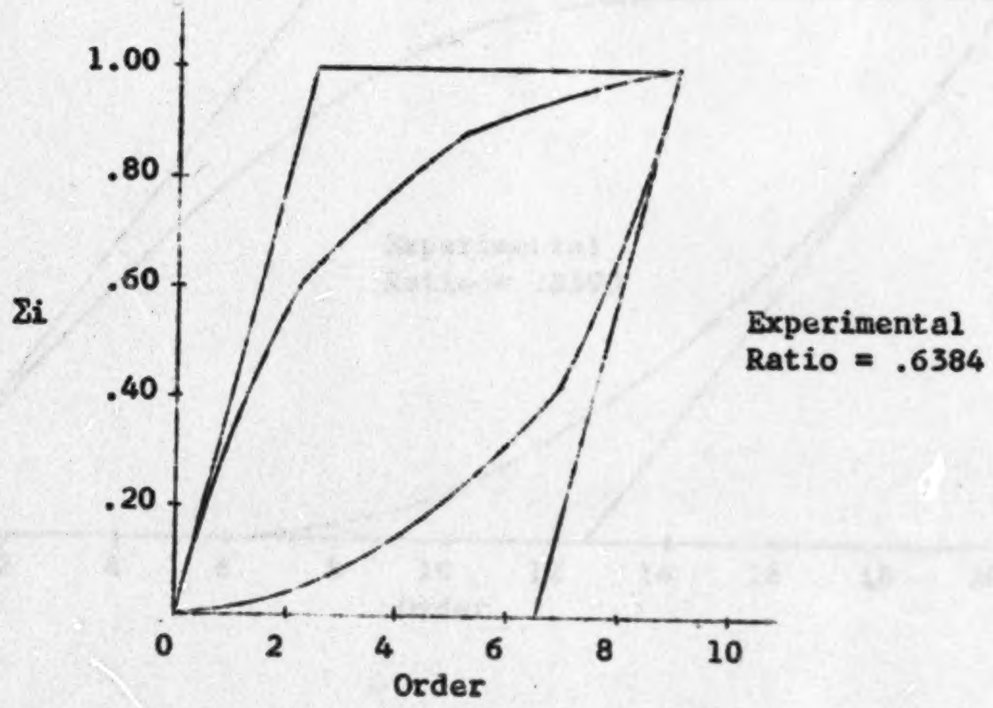
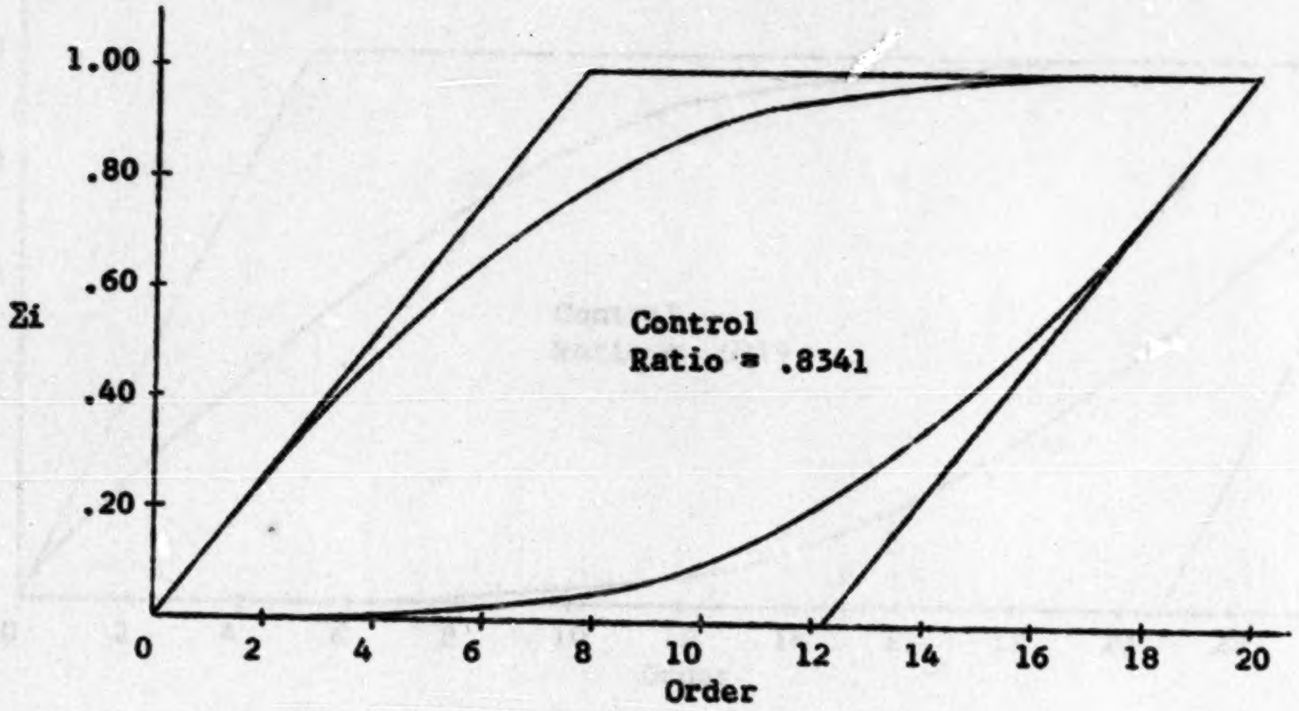


Figure 28. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 1 and Values of Ratios.

Appendix II



Chi Square = .3071

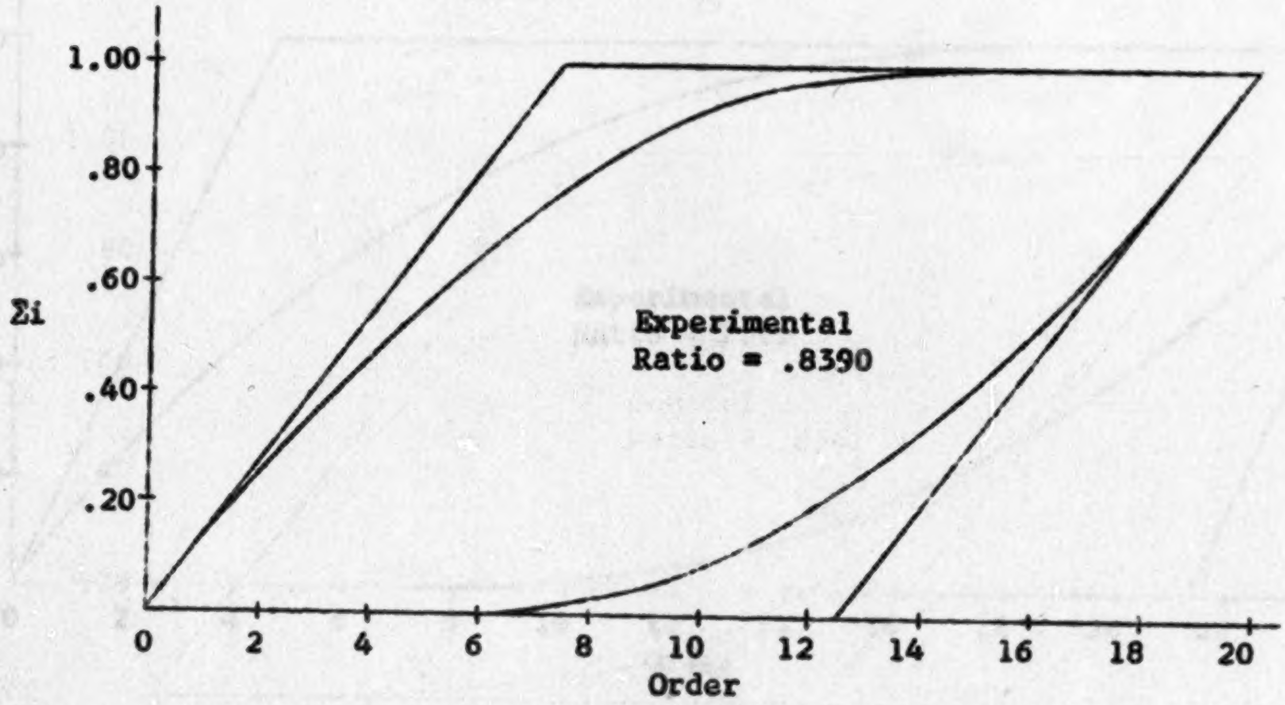
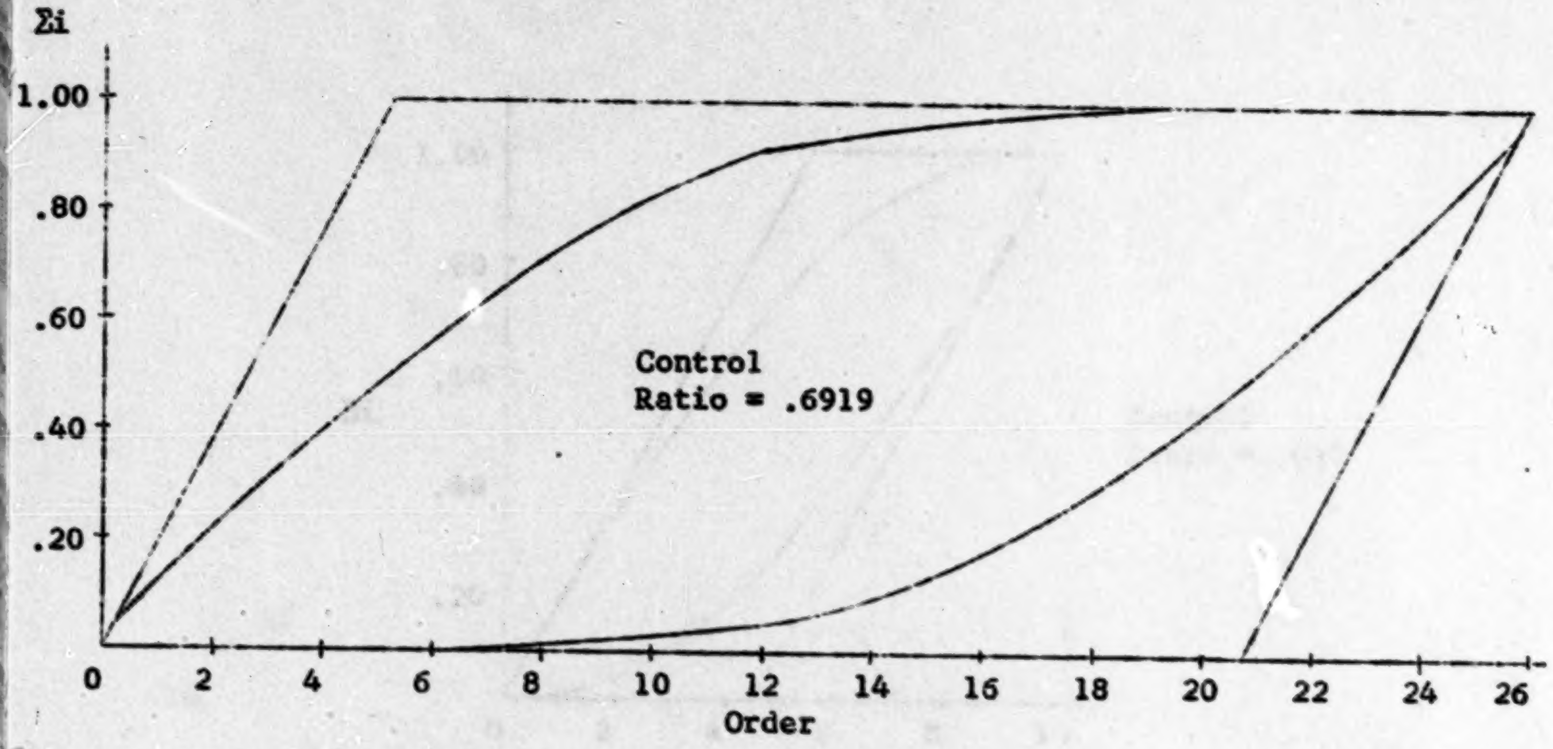


Figure 29. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 19 and Values of Ratios.

Appendix II



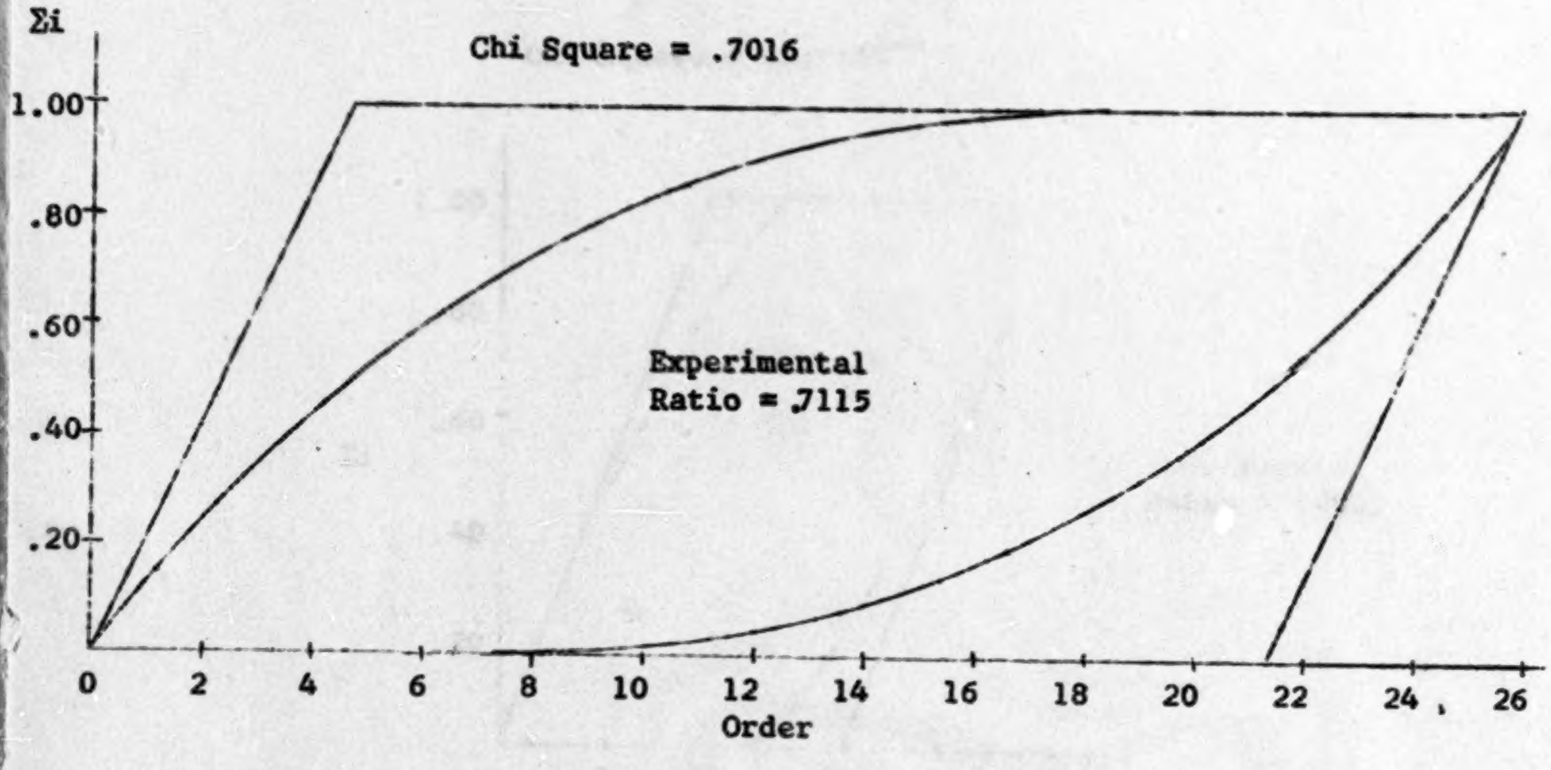
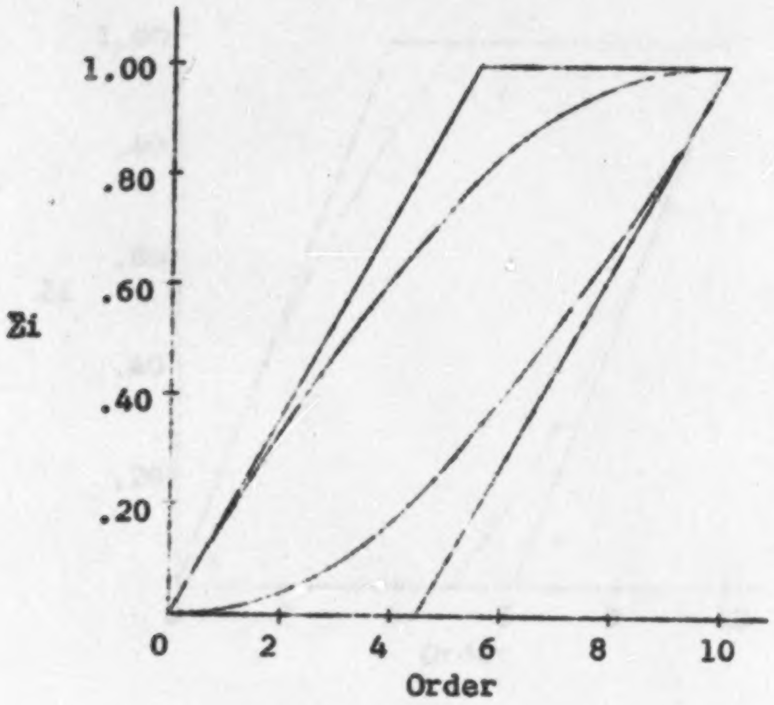


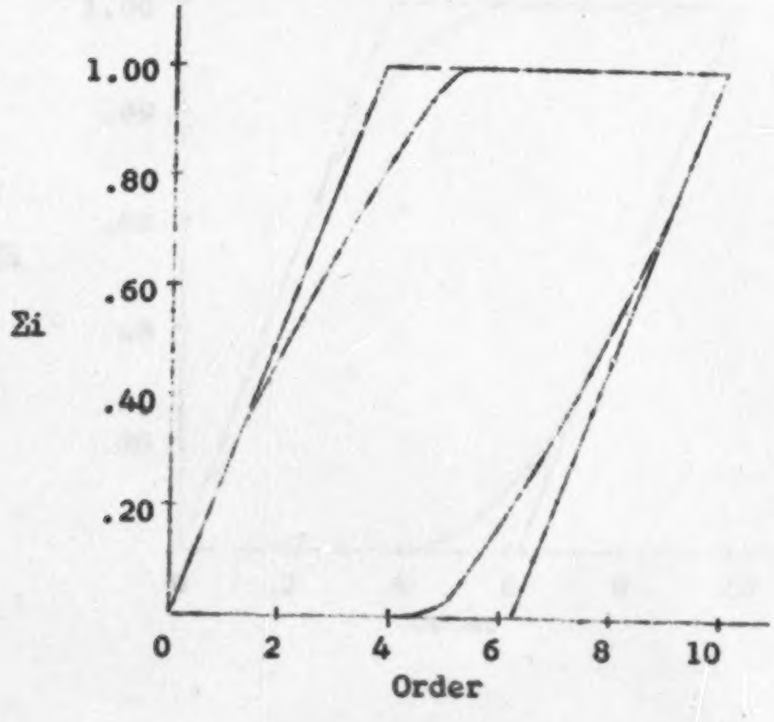
Figure 30. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 25 and Values of Ratios.

Appendix II



Control
Ratio = .699

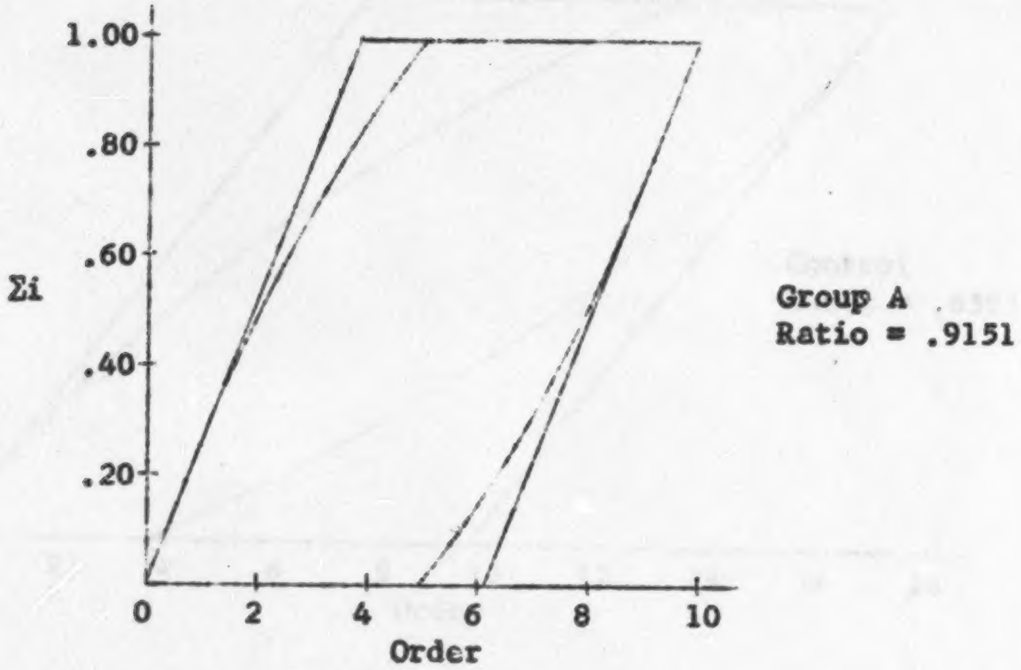
Chi Square = 12.5509^{++}



Experimental
Ratio = .9021

Figure 31. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 31D' and Values of Ratios.

Appendix II



Chi Square = .2629

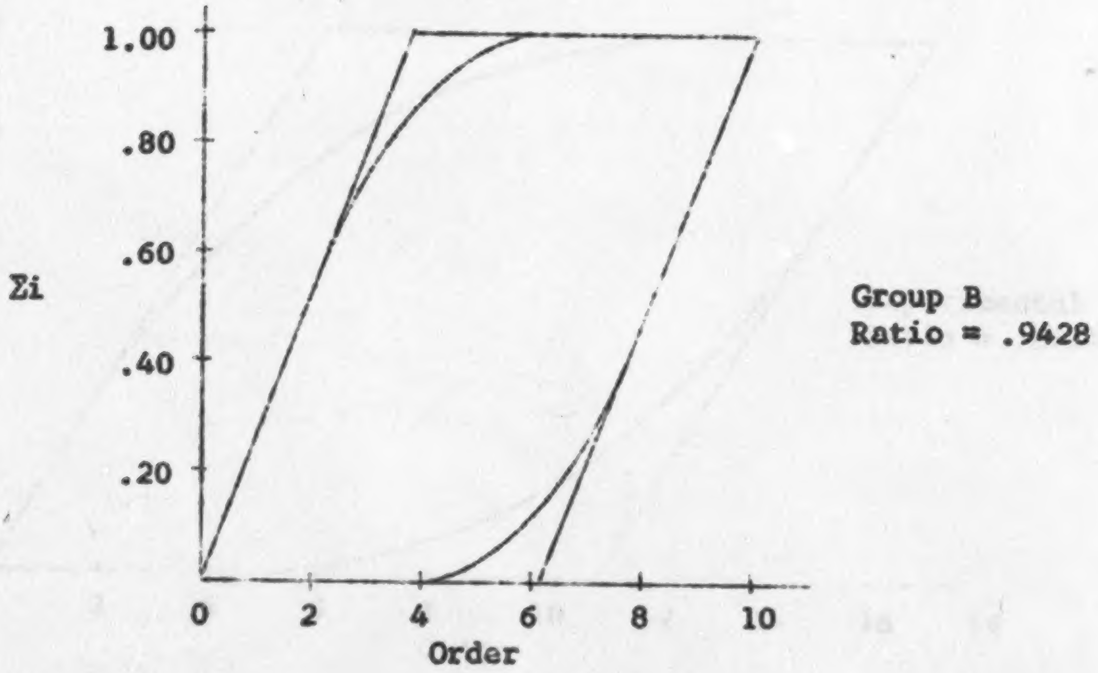
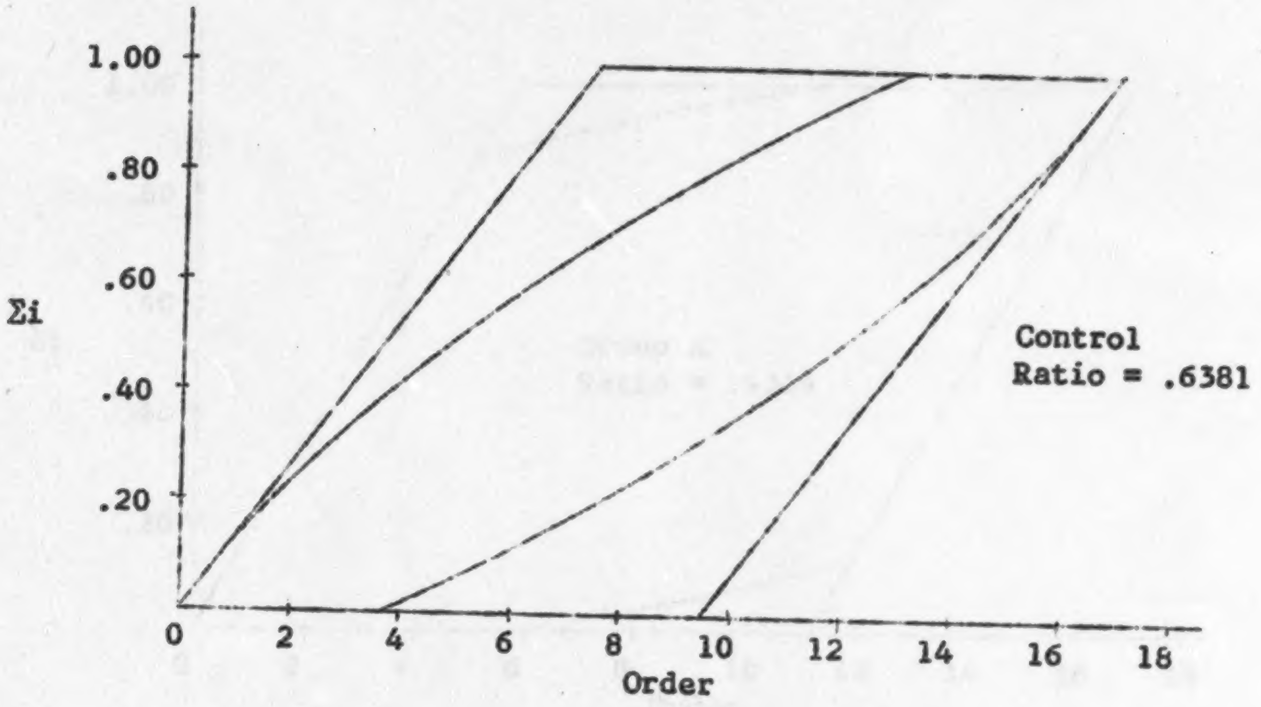


Figure 32. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Post-Training Problem 31D' and Values of Ratios.

Appendix II



Chi Square = 10.9836⁺⁺

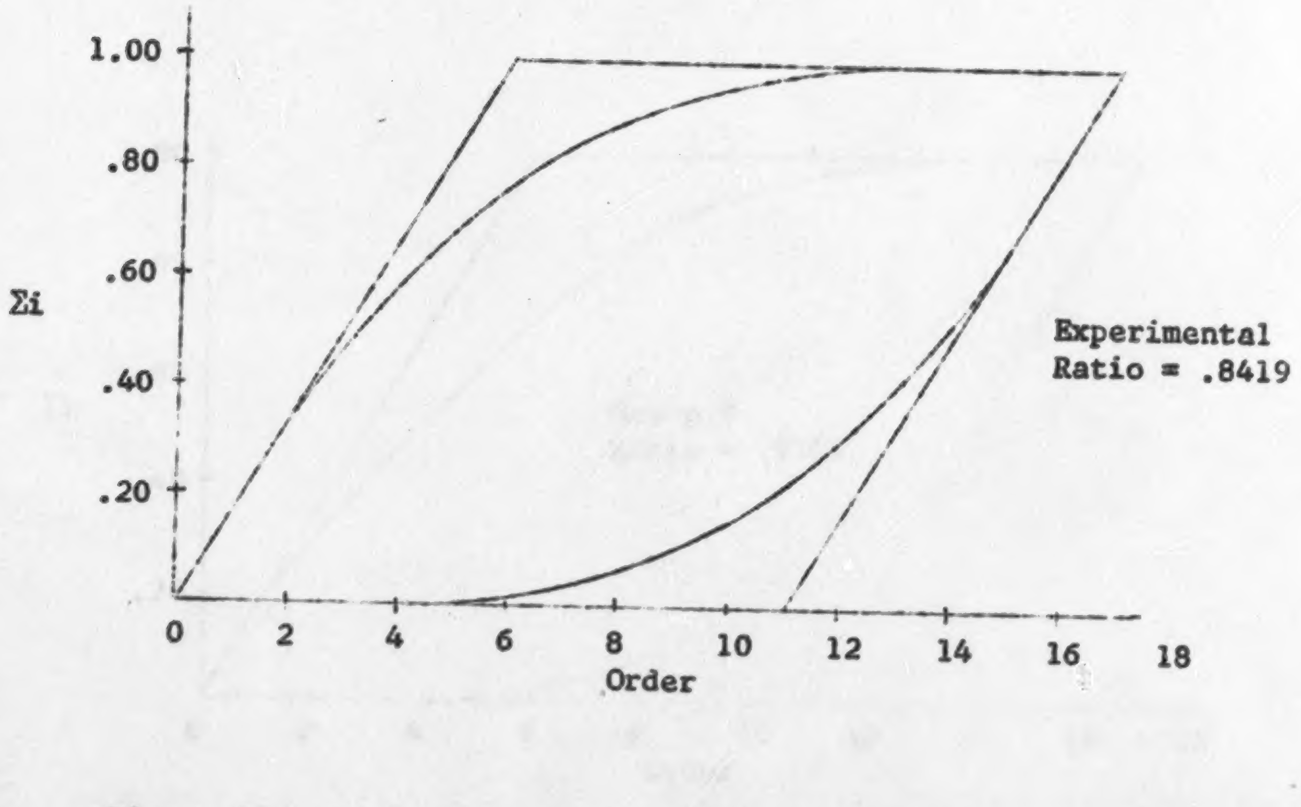
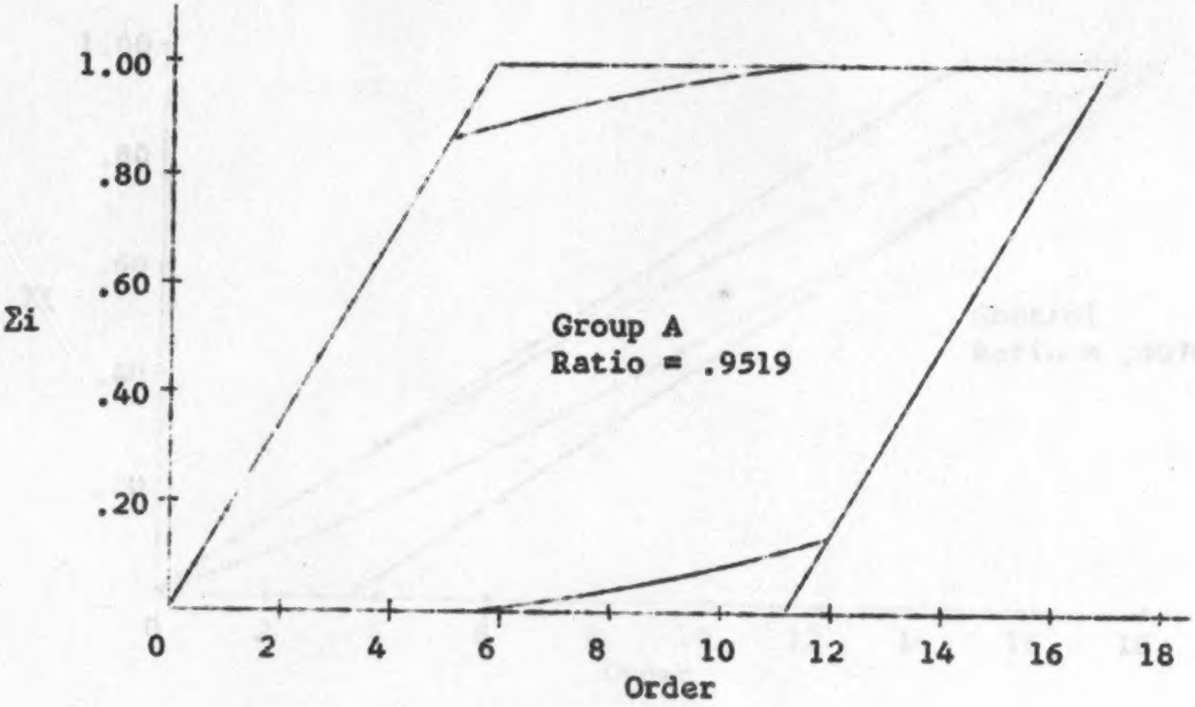


Figure 33. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 35B' and Values of Ratios.

Appendix II



Chi Square = 6.0562^{+}

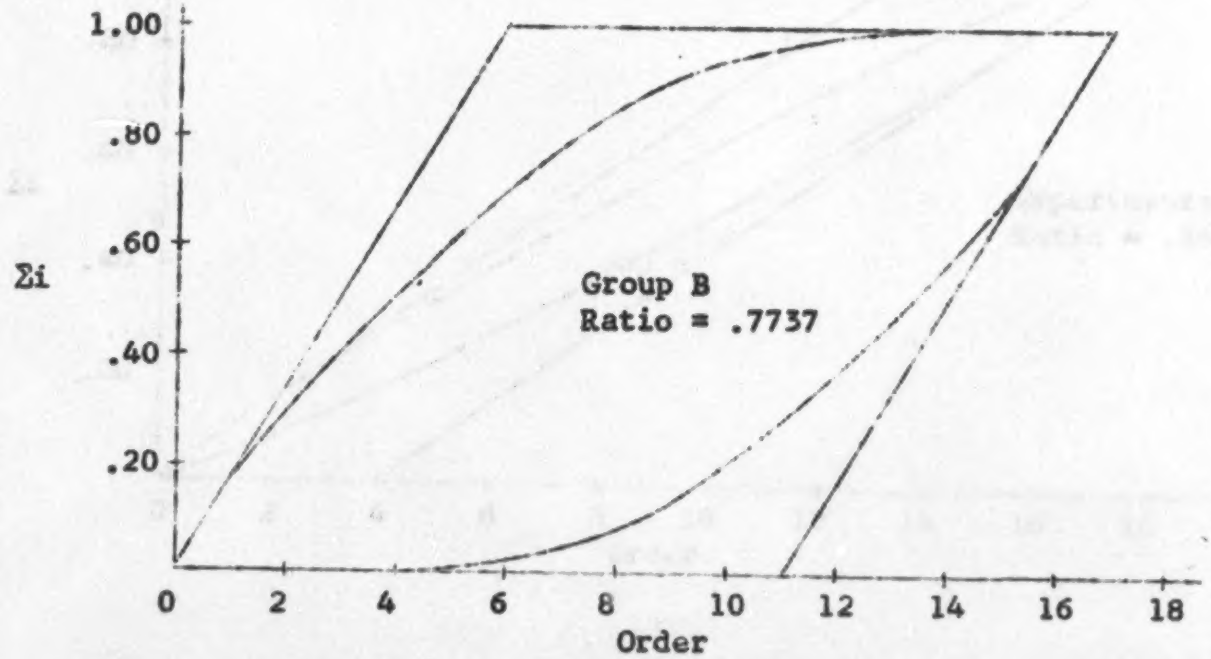
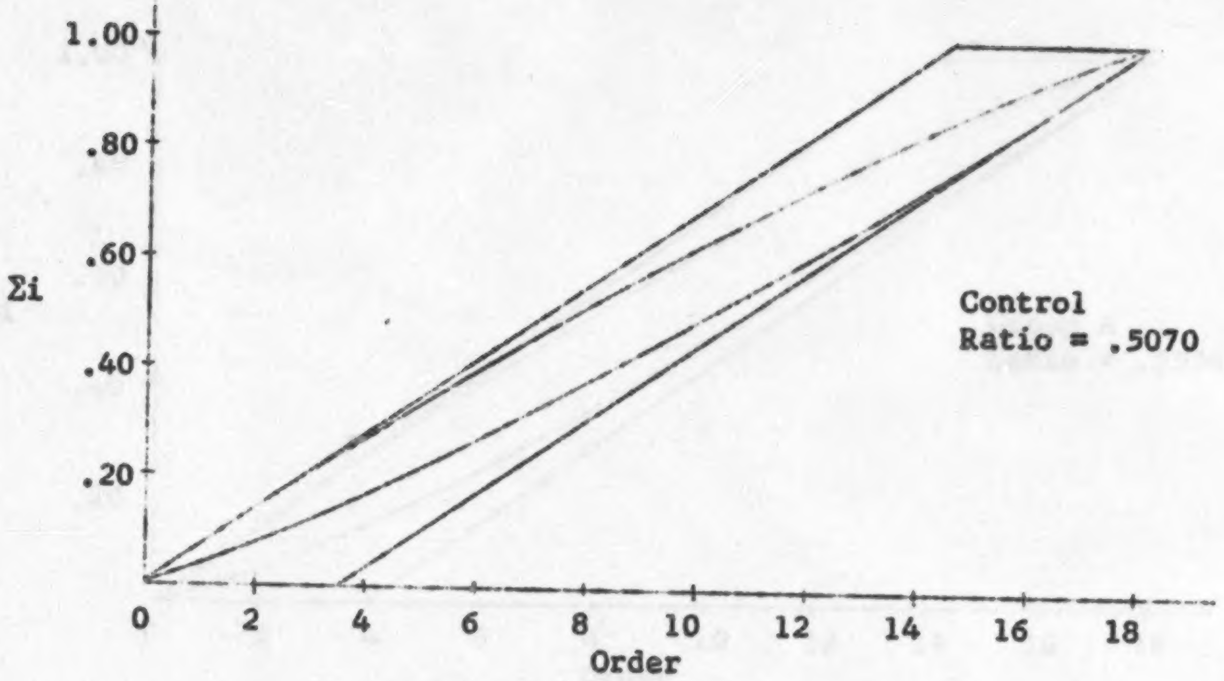


Figure 34. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Post-Training Problem 35B' and Values of Ratios.

Appendix II



Chi Square = .0136

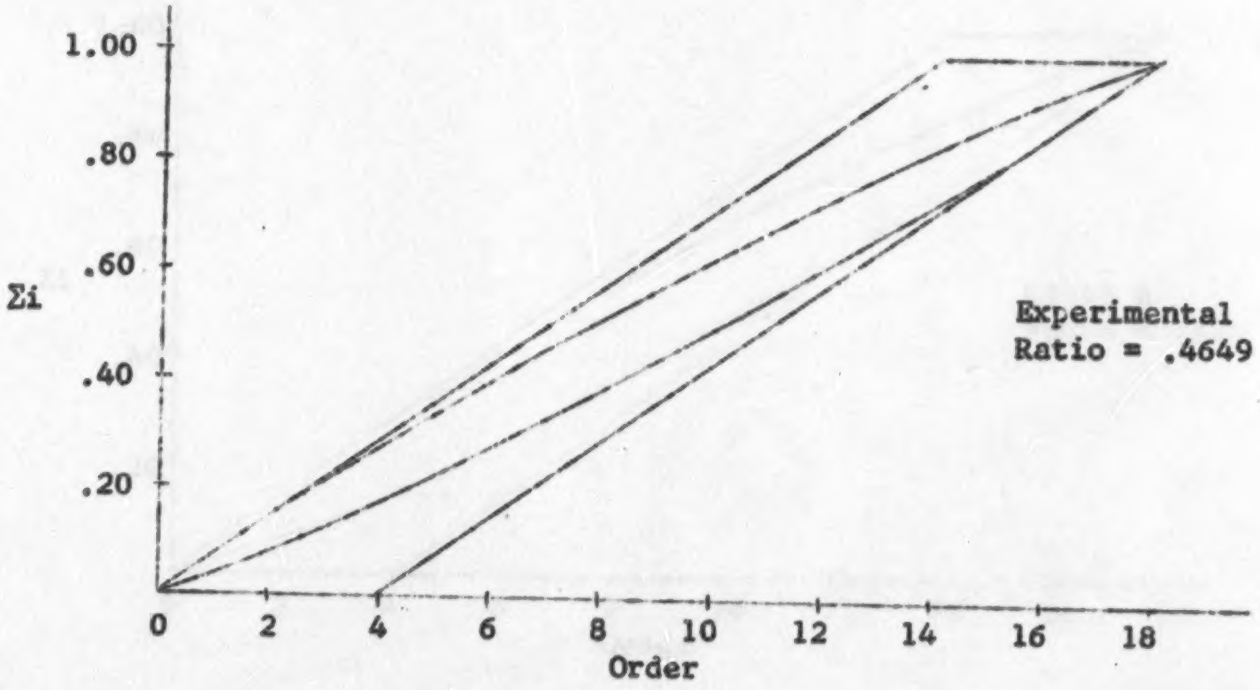
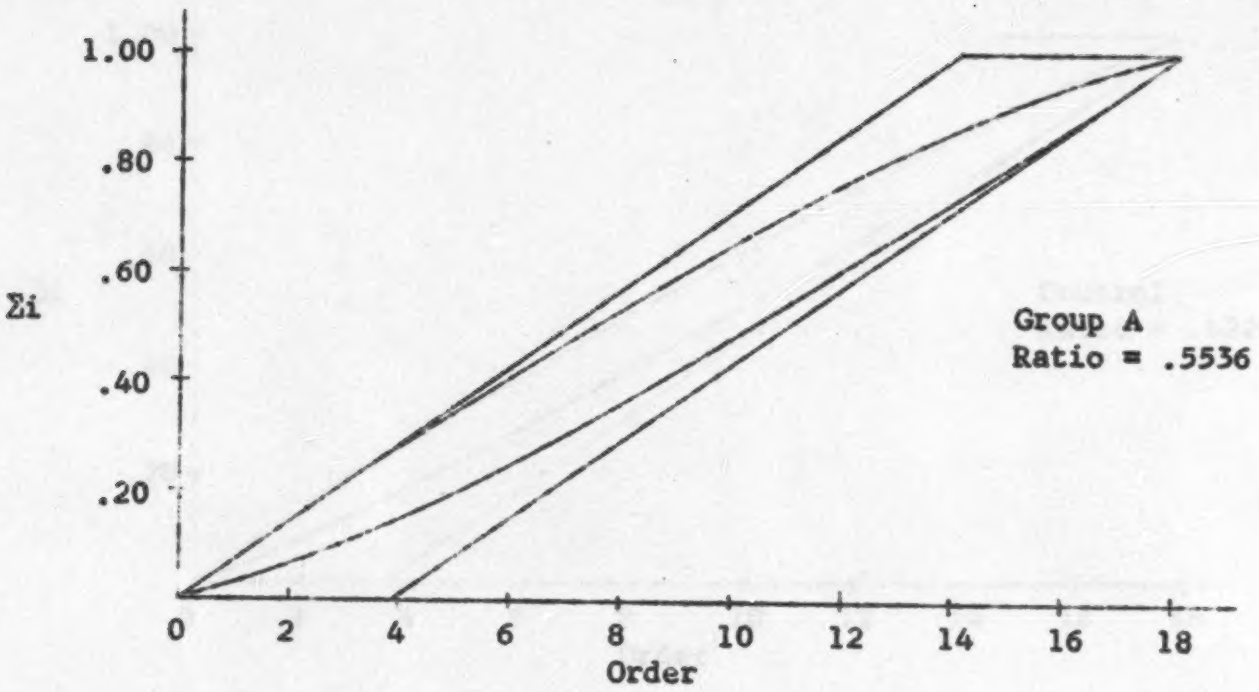


Figure 35. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 32F and Values of Ratios.

Appendix II



Chi Square = .0533

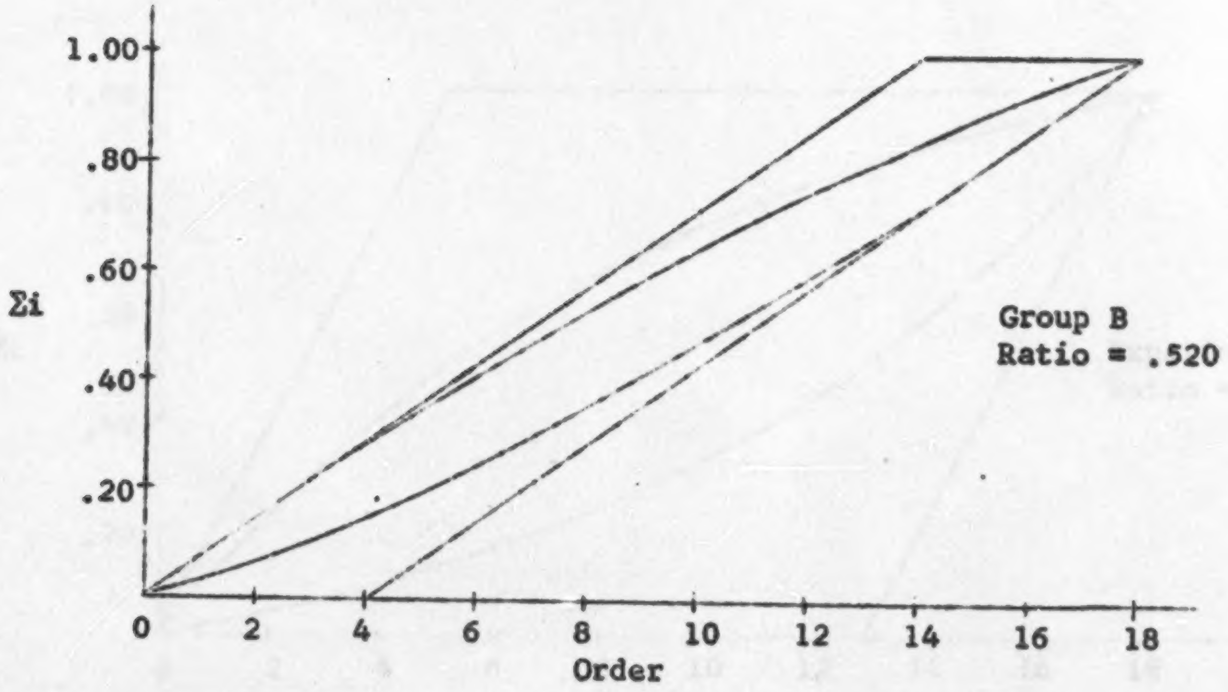
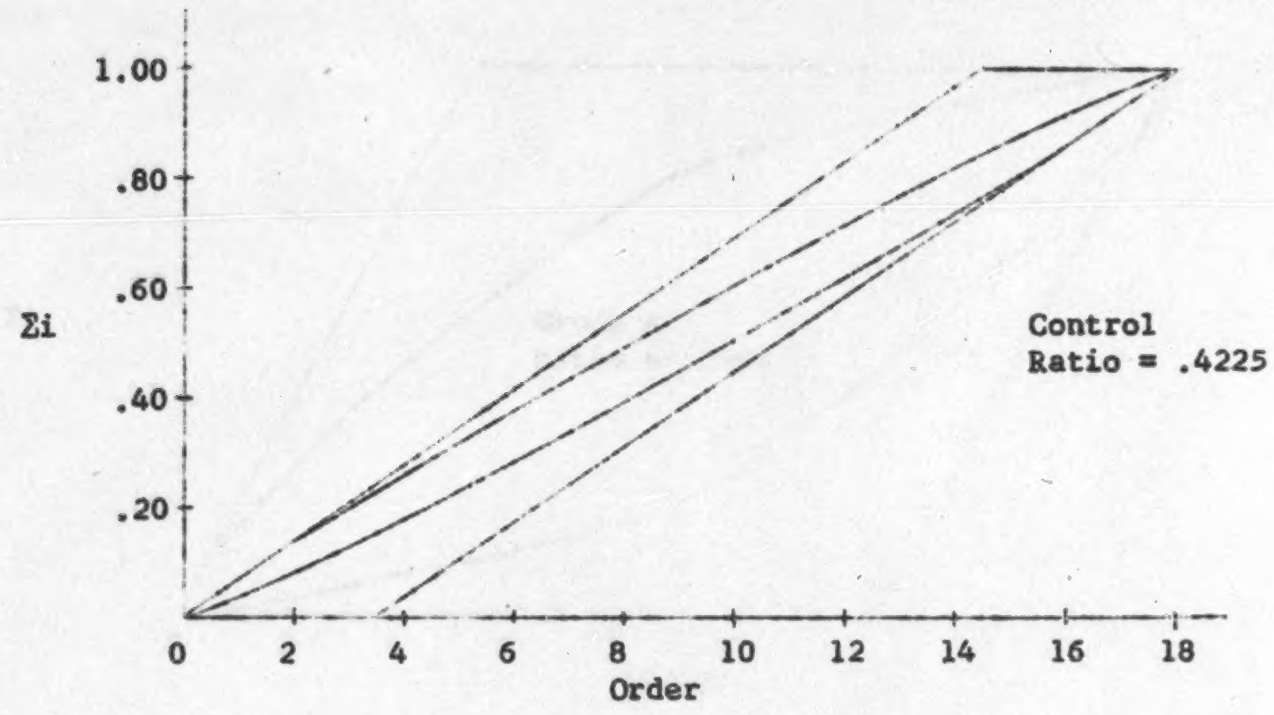


Figure 36. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Post-Training Problem 32F and Values of Ratios.



Chi Square = 10.0889^{++}

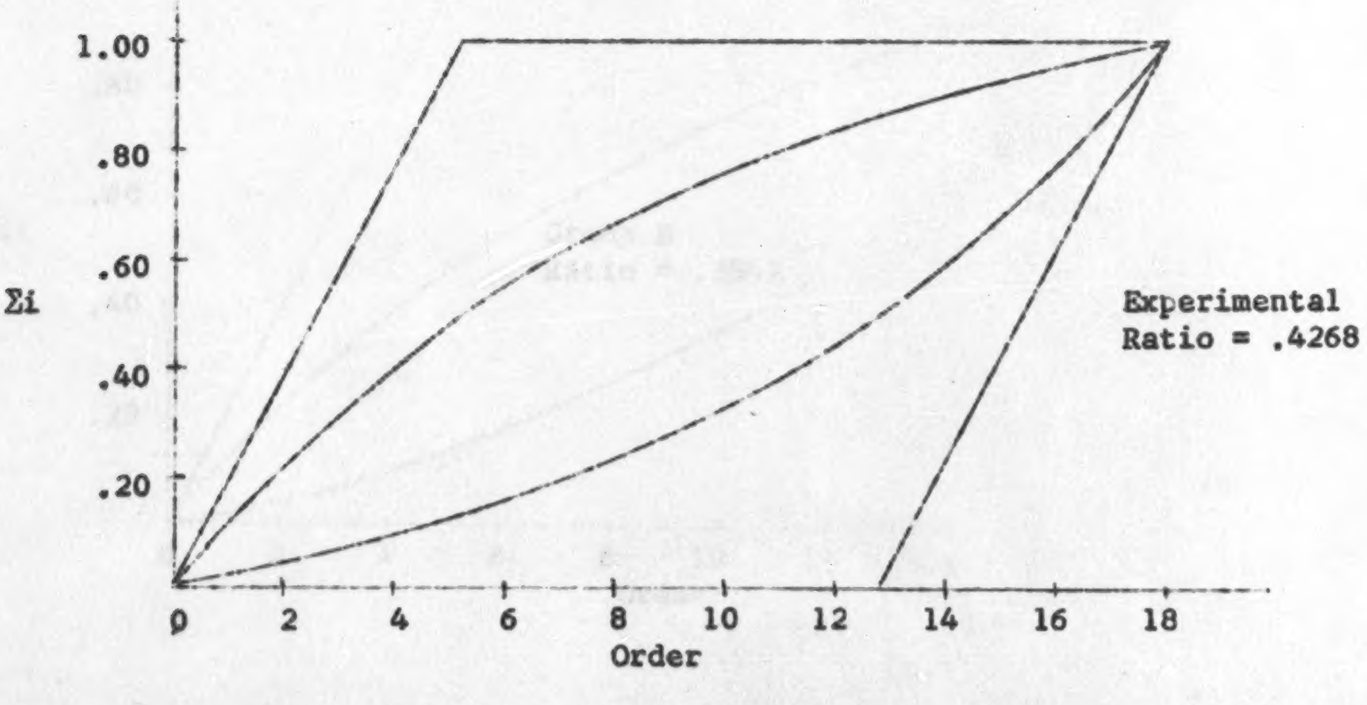
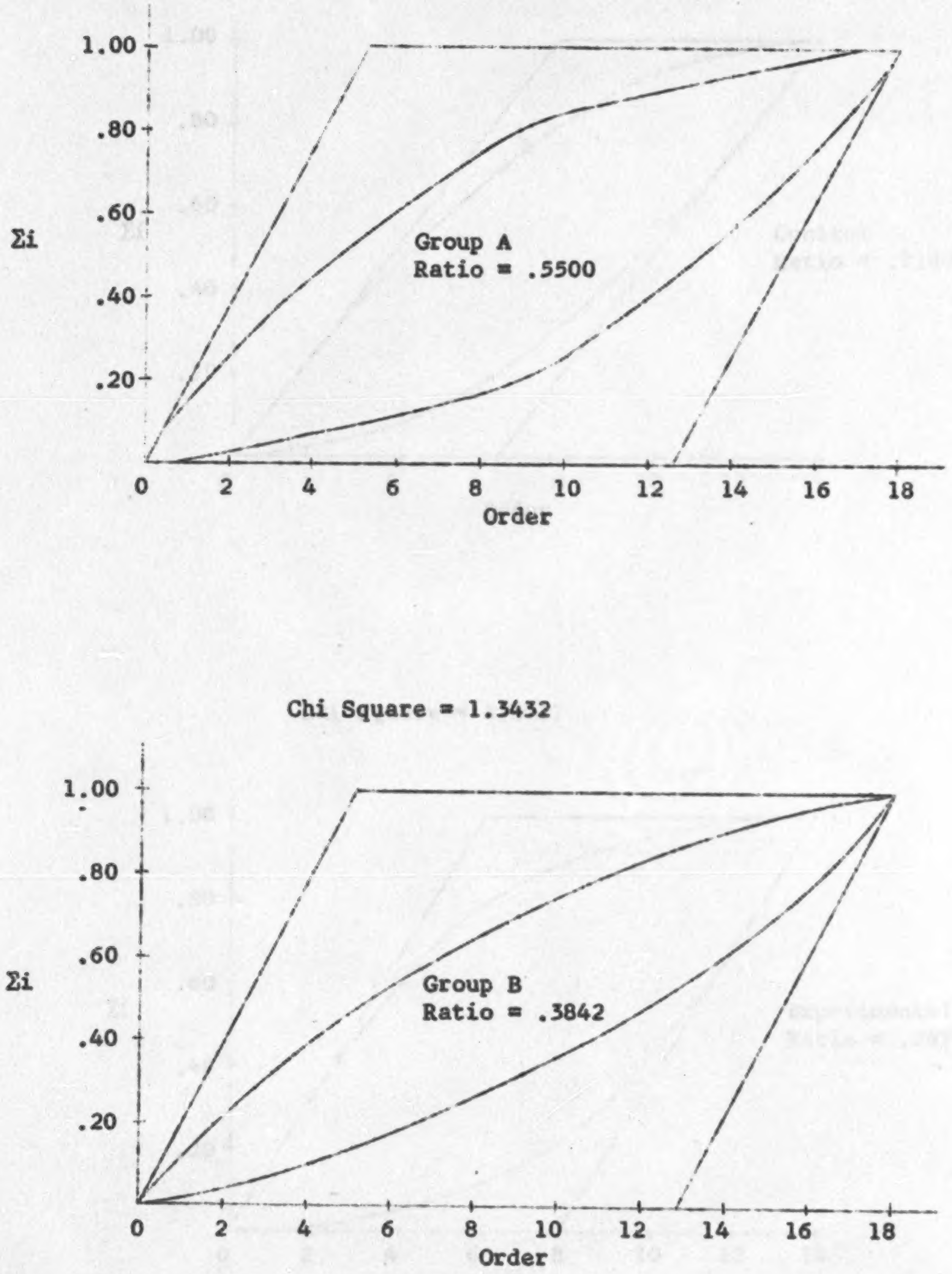


Figure 37. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 36F and Values of Ratios.

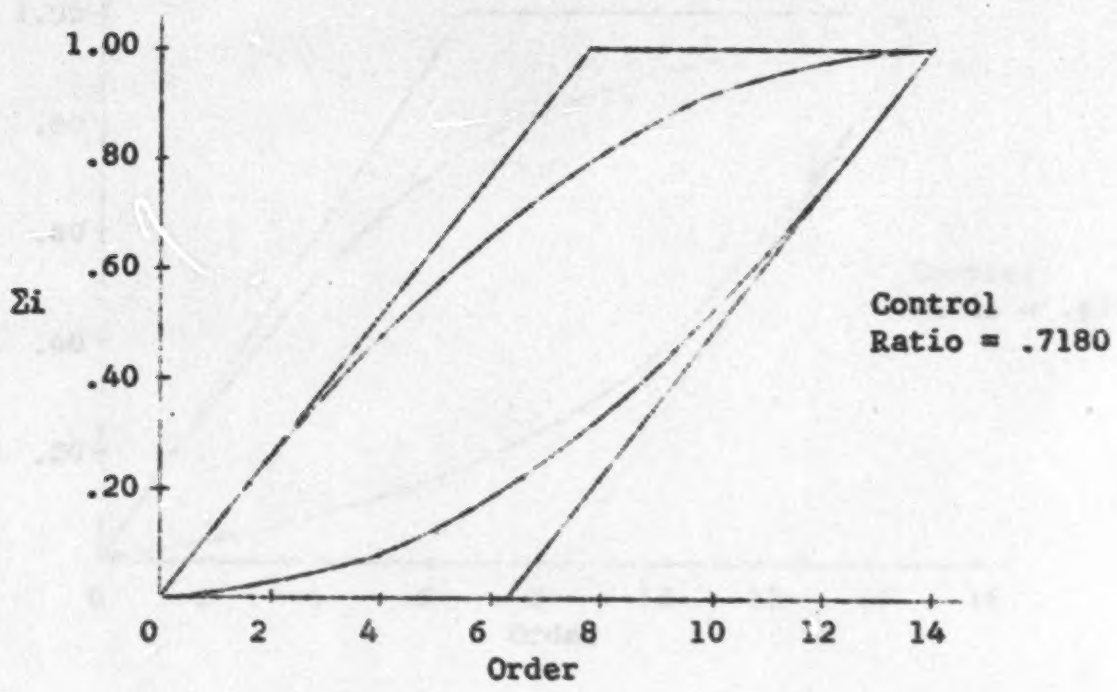
Appendix II



Chi Square = 1.3432

Figure 38. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Experimental Groups A and B on Post-Training Problem 36F and Values of Ratios.

Appendix II



Chi Square = 3.8017

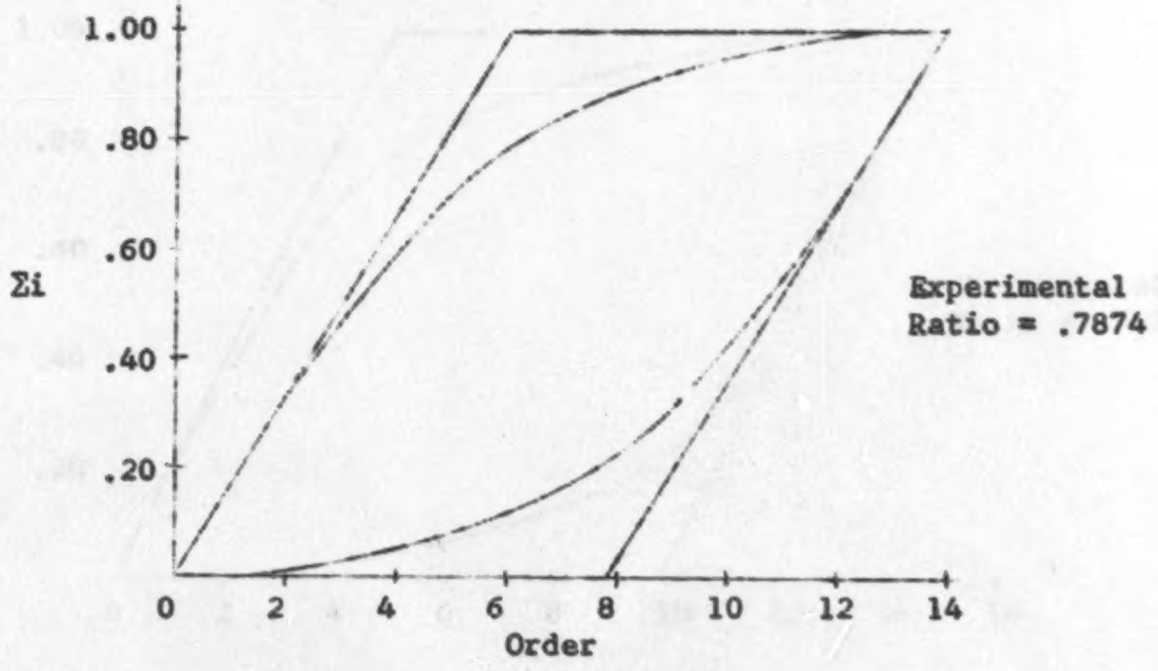
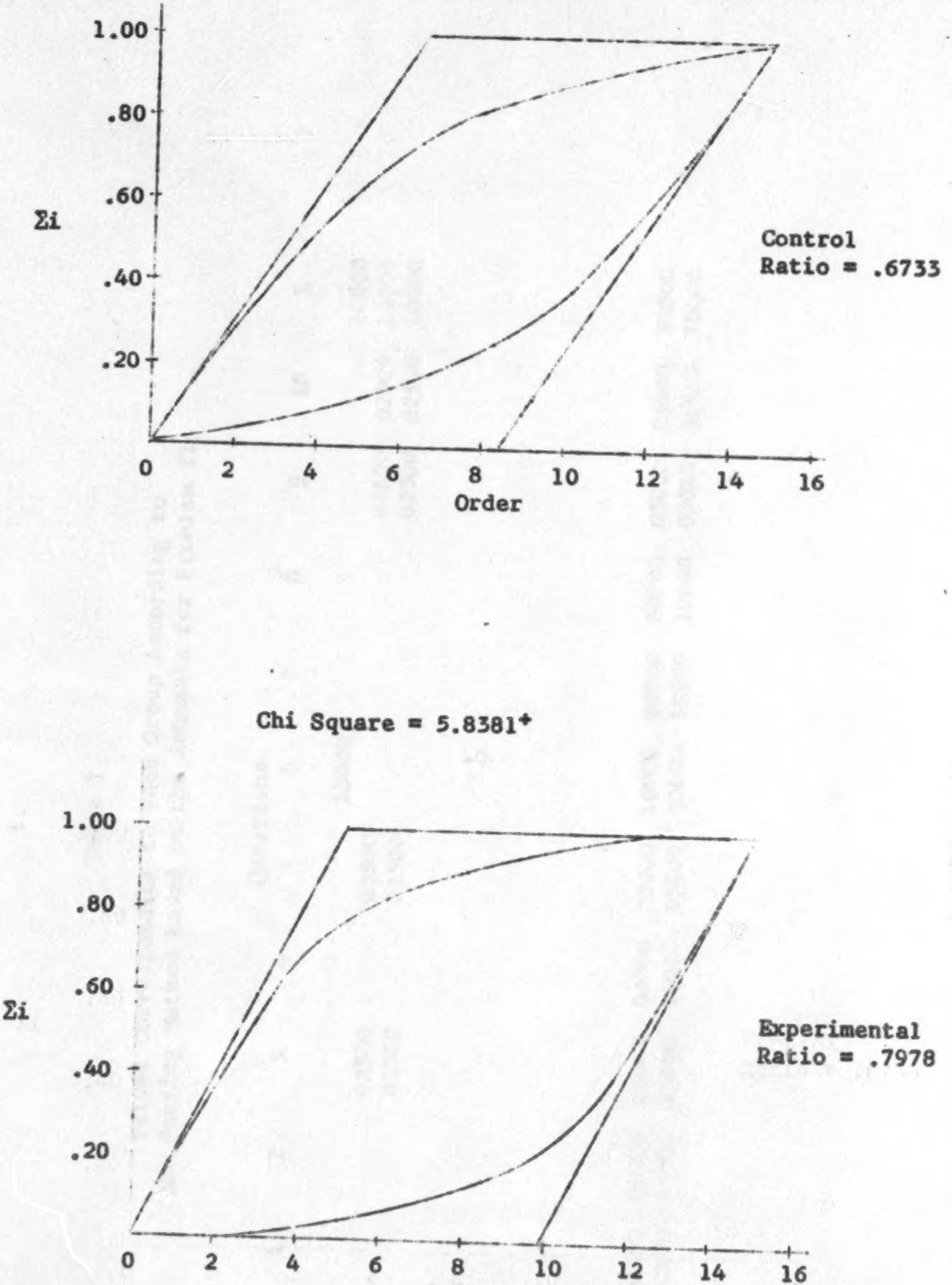


Figure 39. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 26 and Values of Ratios.

Appendix II



Chi Square = 5.8381⁺

Figure 40. Ellipsoids and Corresponding Parallelograms Generated by Performances of Loyola Control and Experimental Subjects on Post-Training Problem 41A and Values of Ratios.

Appendix II

1.

Table 1

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 31A

	Questions										
Order	1	2	3	4	5	6	7	8	9	10	Σ
1						10000					10000
2			02500		02500				02500	02500	10000
3			02500		02500				02500	02500	10000
4											
5											
6											
7											
8											
9											
10											
Σ	00000	00000	05000	00000	05000	10000	00000	00000	05000	05000	30000
Σ_o	10000	10000	05000	10000	05000	00000	10000	10000	05000	05000	70000

Appendix III

Table 2

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 31B

	Questions										
Order	1	2	3	4	5	6	7	8	9	10	Σ
1	10000										10000
2							10000				10000
3		02500		02500				02500	02500		10000
4		02500		02500				02500	02500		10000
5											
6											
7											
8											
9											
10											
Σ	10000	05000	00000	05000	00000	00000	10000	05000	05000	00000	40000
Σ_o	00000	05000	10000	05000	10000	10000	00000	05000	05000	10000	60000

Table 3

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 31C

	Questions										
Order	1	2	3	4	5	6	7	8	9	10	Σ
1					10000						10000
2				02500		02500			02500	02500	10000
3				02500		02500			02500	02500	10000
4											
5											
6											
7											
8											
9											
10											
Σ	00000	00000	00000	05000	10000	05000	00000	00000	05000	05000	30000
Σ_o	10000	10000	10000	05000	00000	05000	10000	10000	05000	05000	70000

Appendix III

Table 4

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 31D

	Questions										
Order	1	2	3	4	5	6	7	8	9	10	Σ
1						10000					10000
2		10000									10000
3			02500	02500			02500	02500			10000
4			02500	02500			02500	02500			10000
5											
6											
7											
8											
9											
10											
Σ	00000	10000	05000	05000	00000	10000	05000	05000	00000	00000	40000
Σ_o	10000	00000	05000	05000	10000	00000	05000	05000	10000	10000	70000

Appendix III

Table 5

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 33A

	Questions													
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
1					04274	03419								
2						03419			01709		00855	00855	00855	
3						00855			01709		01709	01709	01709	
4									04274		00855	00855	01709	
5														
6														
7														
8														
9														
10														
11														
12														
13														
Σ	00000	00000	00000	00000	04274	07693	00000	00000	07692	00000	03419	03419	04273	30770
Σ_o	07692	07692	07692	07692	03418	00000	07692	07692	00000	07692	04273	04273	03419	69227

Table 6

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 33B

	Questions													
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
1				03419	04274									
2				03419			01709			00855	00855		00855	
3				00855			01709			01709	01709		01709	
4							04274			00855	01709		00855	
5														
6														
7														
8														
9														
10														
11														
12														
13														
Σ	00000	00000	00000	07693	04274	00000	07692	00000	00000	03419	04273	00000	03419	30770
Σ_o	07692	07692	07692	00000	03418	07692	00000	07692	07692	04273	03419	07692	04273	69227

Appendix III

Table 7

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 33C

	Questions													
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
1		04274							03419					
2						01709			03419	00855	00855	00855		
3						01709			00855	01709	01709	01709		
4						04274				01709	00855	00855		
5														
6														
7														
8														
9														
10														
11														
12														
13														
Σ	00000	04274	00000	00000	00000	07692	00000	00000	07693	04273	03419	03419	00000	30770
Σ_o	07692	03418	07692	07692	07692	00000	07692	07692	00000	03419	04273	04273	07692	69227

Appendix III

Table 8

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 33D

	Questions													
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
1					05494	01099							01099	
2						04396					01099	01099	01099	
3											02198	02198	01099	
4						02198					01099	01099		
5														
6														
7														
8														
9														
10														
11														
12														
13														
Σ	00000	00000	00000	00000	05494	07693	00000	00000	00000	00000	04396	04396	03297	25276
Σ_o	07692	07692	07692	07692	02198	00000	07692	07692	07692	07692	03296	03296	04395	74721

Appendix III

Table 9

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 35A

	Questions																	
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Σ
1				0588														0588
2		0147				0147				0147				0147				0588
3		0147				0147				0147				0147				0588
4			0147				0147								0147	0147		0588
5			0147				0147								0147	0147		0588
Σ	0000	0294	0294	0588	0000	0294	0294	0000	0000	0294	0000	0000	0000	0294	0294	0294	0000	2940
Σ_o	0588	0294	0294	0000	0588	0294	0294	0588	0588	0294	0588	0588	0588	0294	0294	0294	0588	7056

Table 10

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 35B

Order	Questions 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Σ
1					0588													0588
2		0147				0147	0147								0147			0588
3		0147				0147	0147								0147			0588
4				0147						0147			0147	0147				0588
5				0147						0147			0147	0147				0588
Σ	0000	0294	0000	0294	0588	0294	0294	0000	0000	0294	0000	0000	0294	0294	0294	0000	0000	2940
Σ_o	0588	0294	0588	0294	0000	0294	0294	0588	0588	0294	0588	0588	0294	0294	0294	0588	0588	7056

Appendix III

Table 11

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 35C

	Questions																	
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Σ
1					0588													0588
2			0147				0147					0147				0147		0588
3			0147				0147					0147				0147		0588
4		0147				0147				0147	0147							0588
5		0147				0147				0147	0147							0588
Σ	0000	0294	0294	0000	0588	0294	0294	0000	0000	0294	0294	0294	0000	0000	0000	0294	0000	2940
Σ_o	0588	0294	0294	0588	0000	0294	0294	0588	0588	0294	0294	0294	0588	0588	0588	0294	0588	7056

Appendix III

Table 12

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 35D

	Questions																	
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Σ
1			0588															0588
2		0147					0147				0147		0147					0588
3		0147					0147				0147		0147					0588
4				0147				0147				0147				0147		0588
5				0147				0147				0147				0147		0588
Σ	0000	0294	0588	0294	0000	0000	0294	0294	0000	0000	0294	0294	0294	0000	0000	0294	0000	2940
Σ_o	0588	0294	0000	0294	0588	0588	0294	0294	0588	0588	0294	0294	0294	0588	0588	0294	0588	7056

Appendix III

Table 13

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 37A

	Questions														
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Σ
1	01607			00803	00178	00535	00267	00267	03214	00178	00089				07138
2	01607			02053	00178	01607	00178	01339		00178					07140
3				01875	00223	02142	00178	02410	00089	00223					07140
4				01071	00044	01383		01339	03214	00089					07140
5					02142	00357	02142			02187			00312		07140
6				00267	02187	00044	02142			02142			00357		07139
7					02187		02142			02142			00044		06515
8													06428		06428
Σ	03214	00000	00000	06071	07142	06071	07053	05357	06517	07142	00089	00000	07142	00000	55798
Σ_o	03928	07142	07142	01071	00000	01071	00089	01785	00625	00000	07053	07142	00000	07142	44190

Appendix III

Table 14

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem 37B

	Questions														
Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Σ
1		01607		00535	00803	00178	03214	00267	00267			00178		00089	07138
2		01607		01607	02053	00178		00178	01339			00178			07140
3				02142	01875	00223	00089	00178	02410			00223			07140
4				01383	01071	00044	03214		01339			00089			07140
5				00357		02142		02142				02187	00312		07140
6				00044	00267	02187		02142				02142	00357		07139
7						02187		02142				02142	00044		06515
8													06428		06428
Σ	00000	03214	00000	06071	06071	07142	06517	07053	05357	00000	00000	07142	07142	00089	55798
Σ_o	07142	03928	07142	01071	01071	00000	00625	00089	01785	07142	07142	00000	00000	07053	44190

Appendix III